



LIMITLESS POTENTIAL

EMBRACE DIFFERENCE

CALCULATION POLICY

2022-2023

Subject Leader: Mrs L Patterson

This calculation policy has been developed and agreed after using the National Curriculum objectives and methods in conjunction to the Maths Hub schemes of learning. This calculation policy sets out the methods that children will be taught and encouraged to use when tackling calculations as part of their daily Maths lessons and during their work in a wide range of cross-curricular and real-life contexts. The policy reflects our belief that the methods taught should make sense to children and should be both efficient and reliable.

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

What makes a mathematician?

A definition of a Mathematician:

- Someone who can see patterns;
- Someone who shows deeper application;
- Someone who identifies and understands the interconnectivity of concepts and demonstrates this through the transfer of skills;
- Someone who is systematic and resilient and can extend their own learning.

At Towngate Primary Academy, we believe that in order to develop confident, competent and resilient young mathematicians, our pupils benefit from following a mathematical journey to further explore and discover mathematical concepts. To ensure consistency across school, our pupils' learning is deepened by following a clear sequence of learning opportunities:



Fluency

One of the three aims of the National Curriculum states that pupils (of all ages, not just primary children) will:

“become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.”

Fluency is more than simply practicing a calculation or following a procedure; fluency relies on demonstrating an efficient approach to mathematics; choosing to use efficient strategy that can be carried out easily and quickly. Fluency demands accuracy and sound understanding of key number facts, concepts and relationships whilst relying on flexibility to understand that maths can be presented in a number of different ways. Fluency demands more of pupils than simply memorising a single procedure – they need to understand *why* they are doing what they are doing and *know when it is appropriate* to use different methods.

A key element of fluency is procedural variation: to be confident mathematicians, pupils need to see maths in a variety of different ways – using key concepts to apply what they already know to new learning.

Reasoning

The second aim of the National Curriculum indicates that pupils will:

“reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.”

Effective reasoning relies on using and applying prior knowledge to a given context or problem. Reasoning requires logical thinking to tackle a challenging concept – allowing time to try different strategies to reach a given outcome. In practice, through reasoning focused lessons, pupils develop their ability to select appropriate strategies to solve a problem, to draw logical conclusions, to develop and explain solutions / methods and to reflect on their own mathematical challenges and successes. In order for our pupils to reason accurately, we use a range of different question stems to promote discussion and discovery. Some of these include: spot the mistake, find the odd one out, what's the same – what's different, convince me etc. When learners are able to explain and justify their learning and mathematical choices, these skills can be transferred into isolated problem solving investigations and opportunities.

Problem Solving

The third aim of the National Curriculum programme of study for mathematics explains that pupils can:

“can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.”

Problem solving allows opportunities for pupils to apply their understanding to isolated concepts. Through effective problem solving, pupils are able to make sense of mathematics through exploring 'real world' contextual problems; by applying knowledge of a given skill, pupils are able to gain a better understanding of the modern world around them. Problem solving allows for pupils to seek solutions, explore patterns, formulate conjectures. Through problem solving, pupils learn to take risks in their learning, to persevere in a task and experience success.

KEY STAGE 1




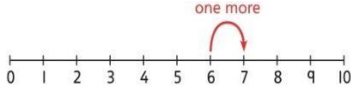
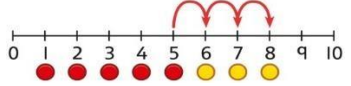

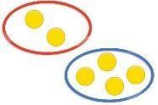
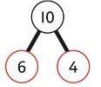

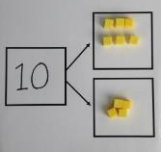
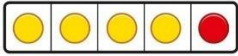
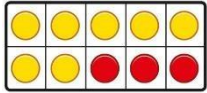
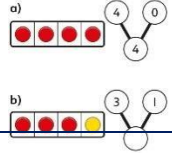
Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with $15 - 3$ and $15 - 13$, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods. In Year 2, they will start to see calculations presented in a column format, although this is not expected to be formalised until KS2. We show the column method in Year 2 as an option; teachers may not wish to include it until Year 3.

Multiplication and division: Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.

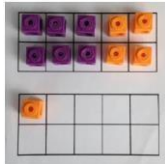
Fractions: In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole. In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.

Year 1

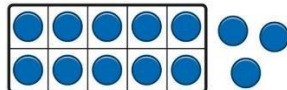
	Concrete	Pictorial	Abstract
Year 1 Addition	<p>Counting and adding more Children add one more person or object to a group to find one more.</p> 	<p>Counting and adding more Children add one more cube or counter to a group to represent one more.</p>   <p>One more than 4 is 5.</p>	<p>Counting and adding more Use a number line to understand how to link counting on with finding one more.</p>  <p>One more than 6 is 7. 7 is one more than 6.</p> <p>Learn to link counting on with adding more than one.</p>  <p>$5 + 3 = 8$</p>
	<p>Understanding part-part-whole relationship (partitioning) Sort people and objects into parts and understand their relationship with the whole.</p>  <p>The parts are 2 and 4. The whole is 6.</p>	<p>Understanding part-part-whole relationship (partitioning) Children draw to represent the parts and understand the relationship with the whole.</p>  <p>The parts are 2 and 4. The whole is 6.</p>	<p>Understanding part-part-whole relationship (partitioning) Use a part-whole model to represent the numbers.</p>  <p>$6 + 4 = 10$</p> <p>$6 + 4 = 10$</p>
	<p>Knowing and finding number bonds within 10 (commutativity) Break apart a group and put back together to find and form number bonds.</p> <p>$6 + 4 = 10$</p>  <p>$6 = 2 + 4$</p> 	<p>Knowing and finding number bonds within 10 (commutativity) Use five and ten frames to represent key number bonds.</p>  <p>$5 = 4 + 1$</p>  <p>$10 = 7 + 3$</p>	<p>Knowing and finding number bonds within 10 (commutativity) Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.</p> <p>$4 + 0 = 4$ $3 + 1 = 4$</p> 

Understanding teen numbers as a complete 10 and some more
Complete a group of 10 objects and count more.

11 is 10 and 1 more.



Understanding teen numbers as a complete 10 and some more
Use a ten frame to support understanding of a complete 10 for teen numbers.

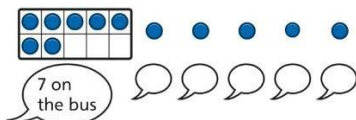


13 is 10 and 3 more.

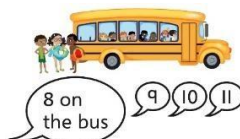
Understanding teen numbers as a complete 10 and some more.

1 ten and 3 ones equal 13.
 $10 + 3 = 13$

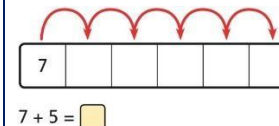
Adding by counting on (augmentation)
Children use counters to support and represent their counting on strategy.



Adding by counting on (augmentation)
Children use knowledge of counting to 20 to find a total by counting on using people or objects.



Adding by counting on (augmentation)
Children use number lines or number tracks to support their counting on strategy.

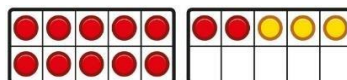


Adding the 1s
Children use concrete materials to recognise how to add the 1s to find the total efficiently.



$2 + 3 = 5$
 $12 + 3 = 15$

Adding the 1s
Children represent calculations using ten frames to add a teen and 1s.



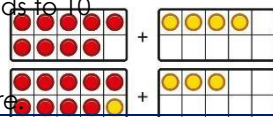
$2 + 3 = 5$
 $12 + 3 = 15$

Adding the 1s
Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently.

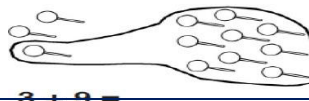
$3 + 5 = 8$
So, $13 + 5 = 18$

Bridging the 10 using number bonds
Children use counters to complete a ten frame and understand how they can add using knowledge of number bonds to 10.

9 add 1 makes 10.
So, 9 add 4 is 10 and 3 more.

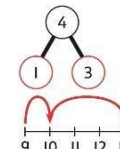


Bridging the 10 using number bonds
Use pictures or a number line. Regroup or partition the smaller number to make 10.



Bridging the 10 using number bonds
Use a part-whole model and a number line to support the calculation.

$9 + 4 = 13$



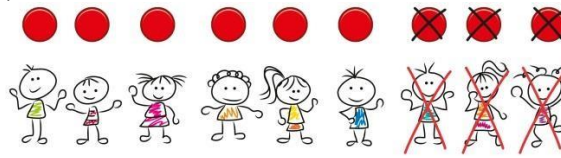
Year 1
Subtraction

Counting back and taking away (reduction) Children arrange objects and remove to find howmany are left.

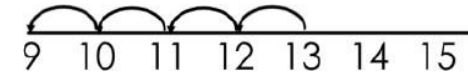


1 less than 6 is 5.
6 subtract 1 is 5.

Counting back and taking away (reduction) Children draw and cross out or use counters to represent objects from a problem.

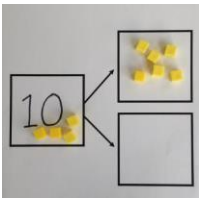


Counting back and taking away (reduction) Children count back to take away and use a numberline or number track to support the method.



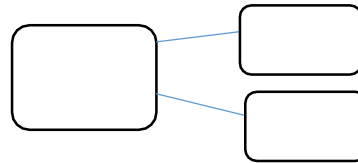
$$9 - 3 = 6$$

Finding a missing part, given a whole and a part (partitioning) Children separate a whole into parts and understand how one part can be found by subtraction.

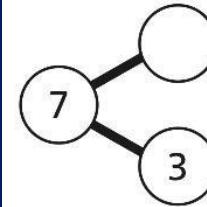


$$10 - 6 = ?$$

Finding a missing part, given a whole and a part (partitioning) Children represent a whole and a part and understand how to find the missing part by subtraction.

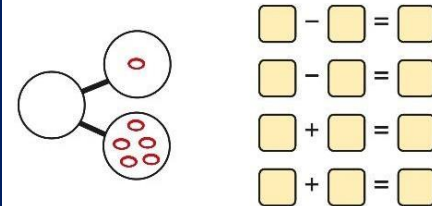


Finding a missing part, given a whole and a part (partitioning) Children use a part-whole model to support the subtraction to find a missing part.



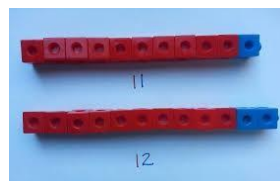
$$7 - 3 = ?$$

Children develop an understanding of the relationship between addition and subtraction facts in a part- whole model.



Finding the difference

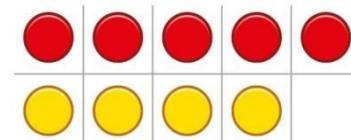
Arrange two groups so that the difference between the groups can be worked out.



12 is 1 more than 11.
11 is 1 less than 12.
The difference between 11 and 12 is 1.

Finding the difference

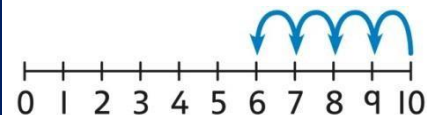
Represent objects using sketches or counters to support finding the difference.



$5 - 4 = 1$
The difference between 5 and 4 is 1.

Finding the difference

Children understand 'find the difference' as subtraction.



$10 - 4 = 6$
The difference between 10 and 6 is 4.

Subtraction within 20

Understand when and how to subtract 1s

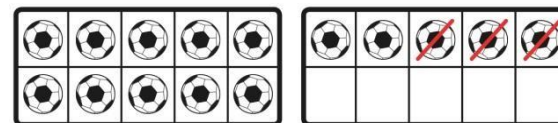
efficiently. Use a bead string to subtract 1s efficiently.



$5 - 3 = 2$
 $15 - 3 = 12$

Subtraction within 20

Understand when and how to subtract 1s efficiently.



$5 - 3 = 2$
 $15 - 3 = 12$

Subtraction within 20

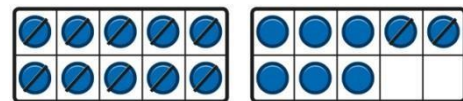
Understand how to use knowledge of bonds within 10 to subtract efficiently.

$5 - 3 = 2$
 $15 - 3 = 12$

Subtracting 10s and 1s

For example: $18 - 12$

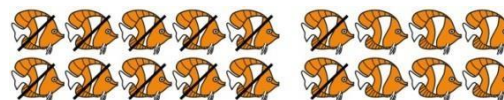
Subtract 12 by first subtracting the 10, then the remaining 2.



First subtract the 10, then take away 2.

Subtracting 10s and 1s

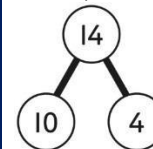
For example: $18 - 12$




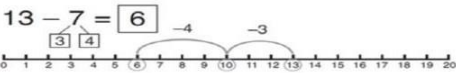
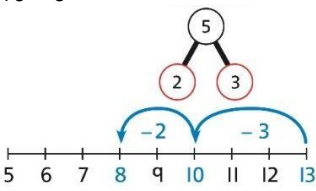
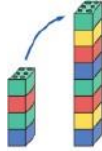




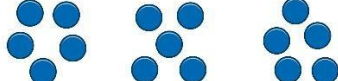
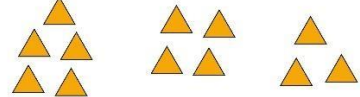
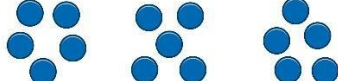
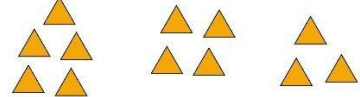
First subtract the 10, then subtract 2.

Subtracting 10s and 1s

Use a part-whole model to support the calculation.



$19 - 14$
 $19 - 10 = 9$
 $9 - 4 = 5$
So, $19 - 14 = 5$

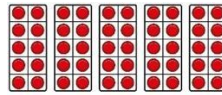
	<p>Subtraction bridging 10 using number bonds For example: $14 - 9$</p> <p>Arrange objects into a 10 and some 1s, then decide on how to split the 9 into parts.</p>  <p>9 is 4 and 5, so I take away the 4 and then the 5.</p>	<p>Subtraction bridging 10 using number bonds Represent the use of bonds using ten frames.</p>  <p>For $13 - 7$, I take away 3 to make 10, then take away 4 to make 6.</p>	<p>Subtraction bridging 10 using number bonds Use a number line and a part-whole model to support the method.</p> <p>$13 - 5$</p> 
<p>Year 1 Multiplication</p>	<p>Doubling Use practical activities to show how to double a number.</p>  <p>double 4 is 8</p>	<p>Doubling Draw pictures to show how to double a number</p> <p>Double 4 is 8</p> 	<p>Double Use known number facts to mentally double numbers within 10.</p> <p>Double 4 = 8</p>
	<p>Recognising and making equal groups Children arrange concrete objects in equal and unequal groups and understand how to recognise whether they are equal.</p> <p>A  B  C </p> <p>A  B </p>	<p>Recognising and making equal groups Children draw and represent equal and unequal groups.</p> <p>A  B </p>	<p>Describe equal groups using words</p> <p>Three equal groups of 4. Four equal groups of 3.</p>
	<p>Finding the total of equal groups by counting in 2s, 5s and 10s</p>	<p>Finding the total of equal groups by counting in 2s, 5s and 10s</p>	<p>Finding the total of equal groups by counting in 2s, 5s and 10s</p>



There are 5 pens in each pack ...
5...10...15...20...25...30...35...40...

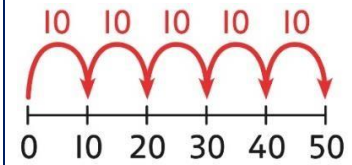


100 squares and ten frames support counting in 2s, 5s and 10s.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

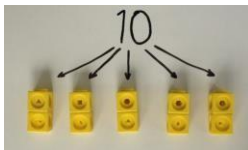
Use a number line to support repeated addition through counting in 2s, 5s and 10s.



Year 1
Division

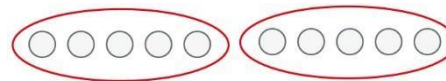
Grouping
Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.

Sort a whole set people and objects into equal groups.



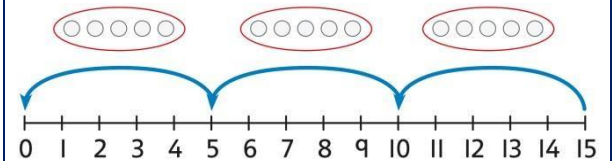
There are 10 children altogether.
There are 5 in each group. There are 2 groups.

Grouping
Represent a whole and work out how many equal groups.

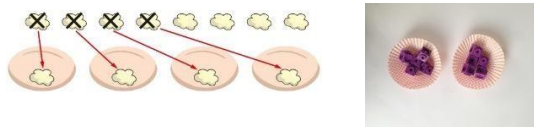


There are 10 in total.
There are 5 in each group. There are 2 groups.

Grouping
Children may relate this to counting back in steps of 2, 5 or 10.



Sharing
Share a set of objects into equal parts and work out how many are in each part.



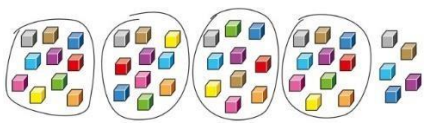

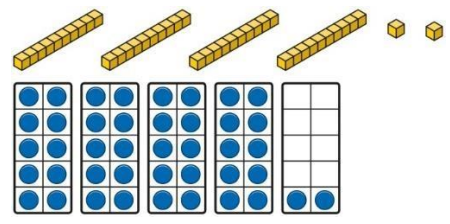
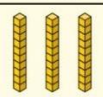

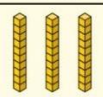

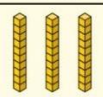


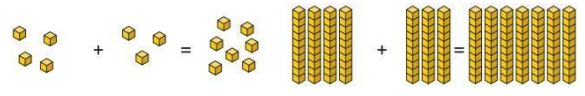
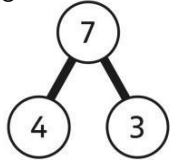


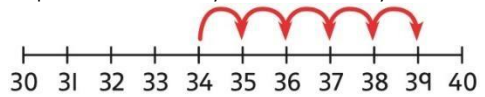
Sharing
Sketch or draw to represent sharing into equal parts. This may be related to fractions.



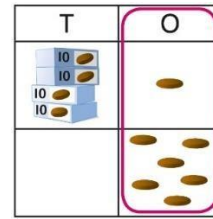
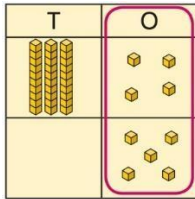
$$8 \div 2 = 4$$

Sharing
10 shared into 2 equal groups gives 5 in each group.

$$10 \div 2 = 5$$

	Concrete	Pictorial	Abstract										
Year 2 Addition													
Understanding 10s and 1s	<p>Group objects into 10s and 1s.</p>  <p>Bundle straws to understand unitising of 10s.</p> 	<p>Understand 10s and 1s equipment, and link with visual representations on ten frames.</p> 	<p>Represent numbers on a place value grid, using equipment or numerals.</p> <table border="1" data-bbox="1590 470 1848 742"> <tr> <th>Tens</th> <th>Ones</th> </tr> <tr> <td></td> <td></td> </tr> <tr> <td>3</td> <td>2</td> </tr> <tr> <th>Tens</th> <th>Ones</th> </tr> <tr> <td>4</td> <td>3</td> </tr> </table>	Tens	Ones			3	2	Tens	Ones	4	3
Tens	Ones												
													
3	2												
Tens	Ones												
4	3												
Adding 10s	<p>Use known bonds and unitising to add 10s.</p>  <p><i>I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.</i></p>	<p>Use known bonds and unitising to add 10s.</p>  <p><i>I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.</i></p>	<p>Use known bonds and unitising to add 10s.</p> <p>$4 + 3 = 7$ $4 \text{ tens} + 3 \text{ tens} = 7 \text{ tens}$ $40 + 30 = 70$</p>  <p>$4 + 3 = \square$</p>										
Adding a 1-digit number to a 2-digit number not bridging a 10	<p>Aggregation and Augmentation Add the 1s to find the total. Use known bonds within 10.</p>  <p><i>34 is 3 tens and 4 additional ones. 34 add 5 ones is 3 tens and 9 additional ones.</i></p>	<p>Aggregation and Augmentation Add the 1s.</p>  <p><i>41 is 4 tens and 1 additional ones. 1 ones and 6 ones are 7 ones. The total is 3 tens and 7 additional ones.</i></p>	<p>Aggregation and Augmentation Add the 1s.</p> <p>Understand the link between counting on and using known number facts. Children should be encouraged to use known number bonds to improve efficiency and accuracy.</p> 										

This can also be done in a place value grid.



This can be represented horizontally or vertically.

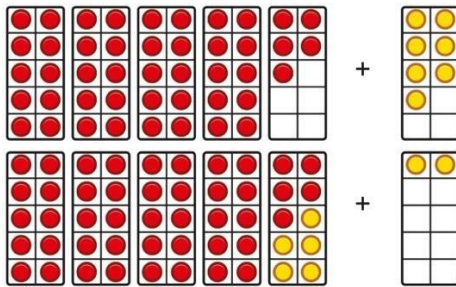
$$34 + 5 = 39$$

or

	T	O
3		4
+		5
—		9
—		—

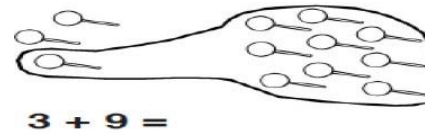
Adding a 1-digit number to a 2-digit number bridging 10

Making 10
Complete a 10 using number bonds.

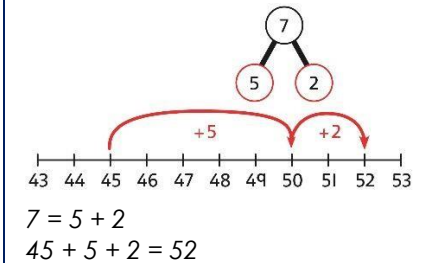


There are 4 tens and 5 additional ones.
I need to add 7. I will use 5 to complete a 10,
then add 2 more ones.

Making 10
Complete a 10 using number bonds.

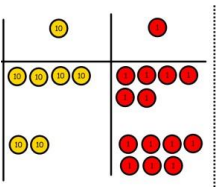


Making 10
Complete a 10 using number bonds.

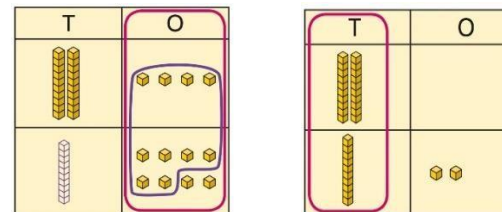


Adding a 1-digit number to a 2-digit number using exchange

Making 10
Exchange 10 ones for 1 ten.



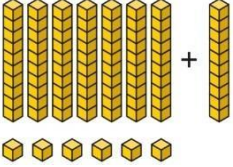


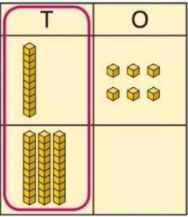
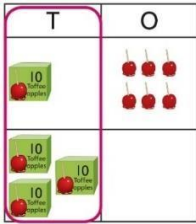
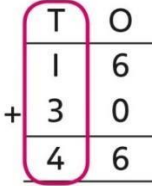
Making 10
Exchange 10 ones for 1 ten.

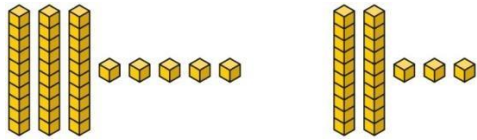


Making 10
Exchange 10 ones for 1 ten.

	T	O
2		4
8		8
+		2
—		—
—		—

	T	O
2		4
8		8
3		2

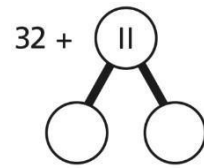
<p>Adding a multiple of 10 to a 2-digit number</p>	<p>Partitioning Add the 10s and then recombine.</p>  <p><i>66 is 6 tens and 6 additional ones.</i> $66 + 10 = 76$</p>	<p>Partitioning Add the 10s and then recombine.</p> <p>A 100 square can support this understanding.</p>   <p><i>27 is 2 tens and 7 additional ones.</i> <i>50 is 5 tens.</i></p> <p><i>There are 7 tens in total and 7 additional ones. So, $27 + 50$ is 7 tens and 7 additional ones.</i></p>	<p>Partitioning Add the 10s and then recombine.</p> <p>$37 + 20 = ?$</p> <p>$30 + 20 = 50$ $50 + 7 = 57$</p> <p>$37 + 20 = 57$</p>
<p>Adding a multiple of 10 to a 2-digit number using columns</p>	<p>Add the 10s using a place value grid to support.</p>  <p><i>16 is 1 ten and 6 additional ones. 30 is 3 tens.</i> <i>There are 4 tens and 6 additional ones in total.</i></p>	<p>Add the 10s using a place value grid to support.</p>  <p><i>16 is 1 ten and 6 additional ones.</i> <i>30 is 3 tens.</i> <i>There are 4 tens and 6 additional ones in total.</i></p>	<p>Add the 10s represented vertically. Children must understand how the method relates to unitising of 10s and place value.</p>  <p>$1 + 3 = 4$ $1 \text{ ten} + 3 \text{ tens} = 4 \text{ tens}$ $16 + 30 = 46$</p>
<p>Adding two 2-digit numbers</p>	<p>Partitioning Add the 10s and 1s separately.</p>	<p>Partitioning Add the 10s and 1s separately. Use a part-whole model to support.</p>	<p>Partitioning Add the 10s and the 1s separately, bridging 10s where required. A number line can support the calculations.</p>



$5 + 3 = 8$
There are 8 ones in total.

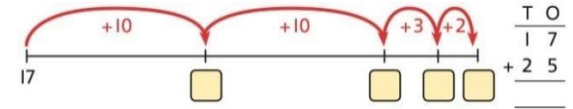
$3 + 2 = 5$
There are 5 tens in total.

$35 + 23 = 58$



$11 = 10 + 1$
 $32 + 10 = 42$
 $42 + 1 = 43$

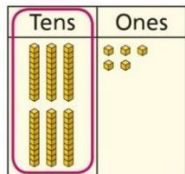
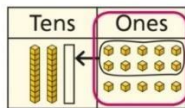
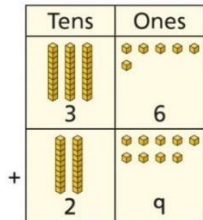
$32 + 11 = 43$



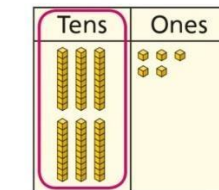
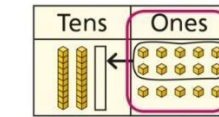
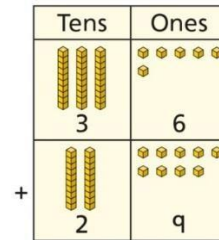
$17 + 25$

Adding two 2-digit numbers with exchange

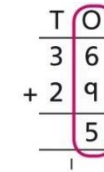
Add the 1s. Emphasis here on children physically exchange 10 ones for a ten. 'One ten is equal to 10 ones' Then add the 10s.



Add the 1s. Exchange 10 ones for a ten. Then add the 10s.



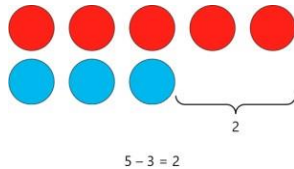
Add the 1s. Exchange 10 ones for a ten. Then add the 10s.



Year 2
Subtraction

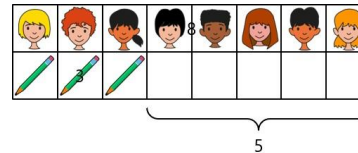
Subtraction
as finding the
difference

Compare the set of objects in each group



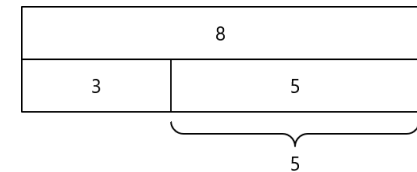
$$5 - 3 = 2$$

Compare the set of objects in each group



$$8 - 3 = 5$$

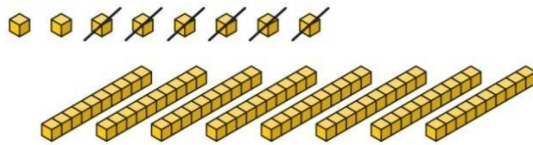
Compare the set of objects in each group



$$8 - 3 = 5$$

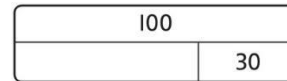
Subtracting
multiples of
10

Use known number bonds and unitising to
subtract multiples of 10.

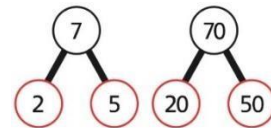


8 subtract 6 is 2.
So, 8 tens subtract 6 tens is 2 tens.

Use known number bonds and unitising to
subtract multiples of 10.



$10 - 3 = 7$
So, 10 tens subtract 3 tens is 7 tens.

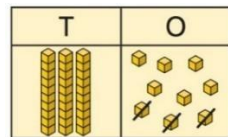
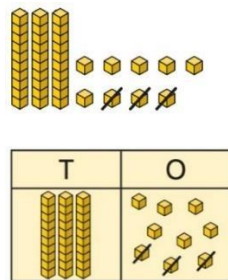


Use known number bonds and unitising to
subtract multiples of 10.

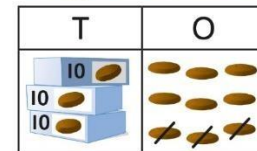
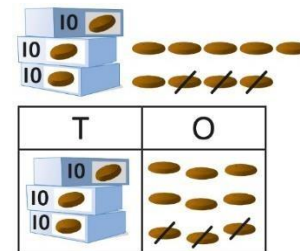
7 tens subtract 5 tens is 2 tens.
 $70 - 50 = 20$

Subtracting a
single-digit
number

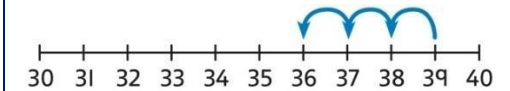
Subtract the 1s. This may be done in or out of a
placevalue grid.



Subtract the 1s. This may be done in or out of a
placevalue grid.



Subtract the 1s. Understand the link between
counting back and subtracting the 1s using known
bonds.

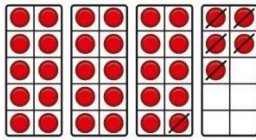


T	O	
3	9	
-	3	
3	6	

$9 - 3 = 6$
 $39 - 3 = 36$

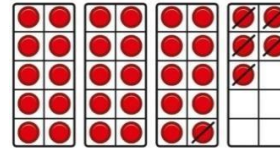
Subtracting a single-digit number bridging 10

Bridge 10 by using known bonds.



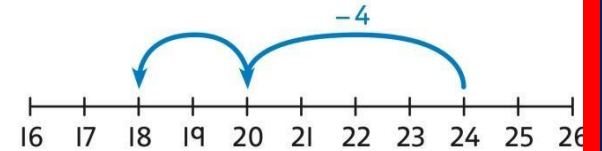
$35 - 6$
*6 is made of 5 ones plus 1 one.
 I took away 5 counters, then 1 more.*

Bridge 10 by using known bonds.
 Drawing representations and crossing out to subtract.



$35 - 6$
First, I will subtract 5, then 1.

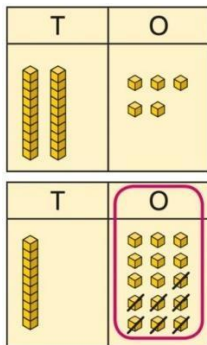
Bridge 10 by using known bonds.



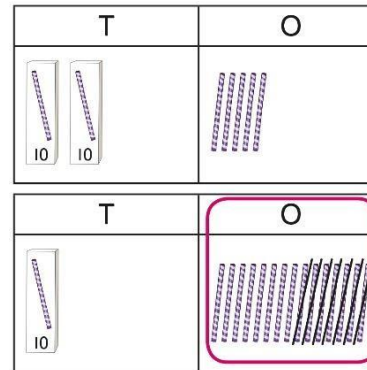
$24 - 6 = ?$
 $24 - 4 - 2 = ?$

Subtracting a single-digit number using exchange

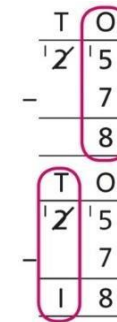
Exchange 1 ten for 10 ones. . Emphasis here on children physically exchange 10 ones for a ten. 'Oneten is equal to 10 ones' This may be done in or out of a place value grid.



Exchange 1 ten for 10 ones.



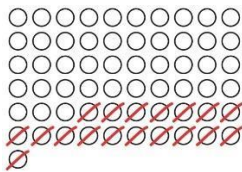
Exchange 1 ten for 10 ones.



$25 - 7 = 18$

Subtracting a 2-digit number

Subtract by taking away.



$61 - 18$
I took away 8 ones and 1 ten.

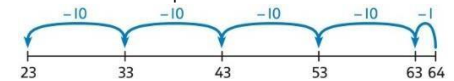
Subtract the 1s and the 10s.

This can be represented on a 100 square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Subtract the 1s and the 10s.

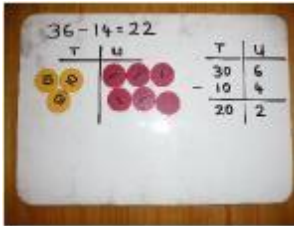
This can be represented on a number line.



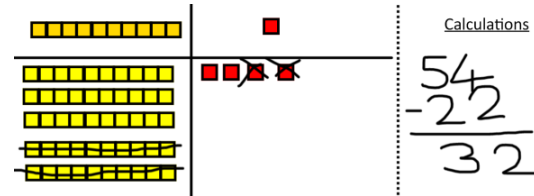
$64 - 41 = ?$
 $64 - 1 = 63$
 $63 - 40 = 23$
 $64 - 41 = 23$

Subtracting a 2-digit number using place value and columns

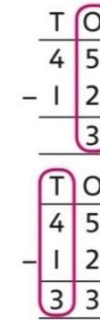
Subtract the 1s. Then subtract the 10s. This may be done in or out of a place value grid.



Subtract the 1s. Then subtract the 10s.

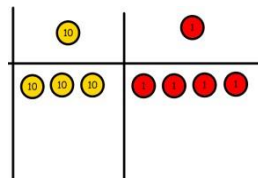


Using column subtraction, subtract the 1s. Then subtract the 10s.

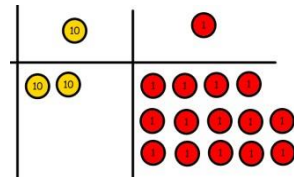


Subtracting a 2-digit number with exchange

Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s. $34 - 18 =$



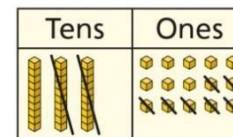
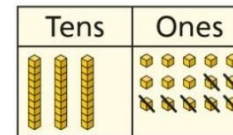
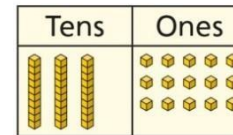
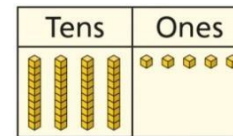
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones



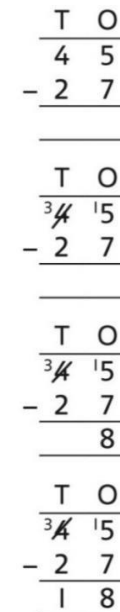
Now I can subtract my ones and my tens.

Exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.



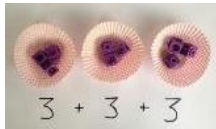
Using column subtraction, exchange 1 ten for 10 ones. Then subtract the 1s. Then subtract the 10s.



Year 2
Multiplication

Equal groups and repeated addition

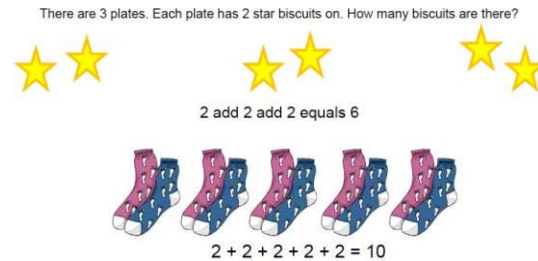
Recognise equal groups and write as repeated addition and as multiplication.



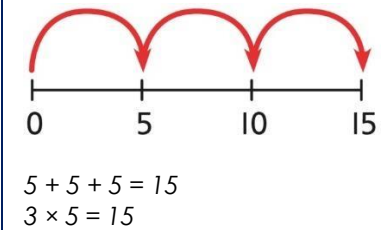
3 groups of 3 sweets
9 sweets altogether



Recognise equal groups using standard objects and write as repeated addition and multiplication.



Use a number line and write as repeated addition and as multiplication.

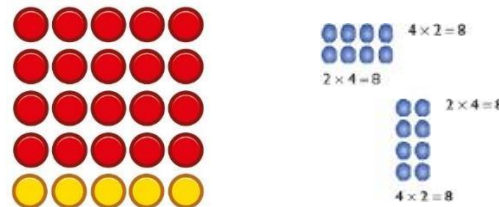


Using arrays to represent multiplication and support understanding

Understand the relationship between arrays, multiplication and repeated addition.

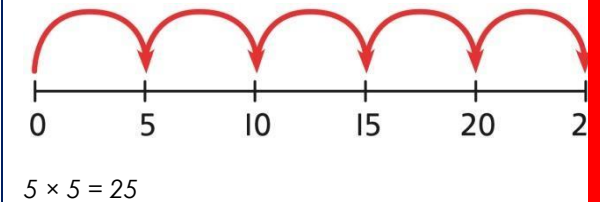
4 groups of 6

Understand the relationship between arrays, multiplication and repeated addition. Draw arrays in different rotations to find commutative multiplication sentences.



4 groups of 5 ... 5 groups of 5

Understand the relationship between arrays, multiplication and repeated addition.

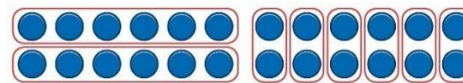


Understanding commutativity

Use arrays to visualise commutativity.

I can see 6 groups of 3. I can see 3 groups of 6.

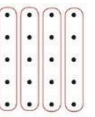
Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.



This is 2 groups of 6 and also 6 groups of 2.

Use arrays to visualise commutativity.

4 + 4 + 4 + 4 + 4 = 20
5 + 5 + 5 + 5 = 20
4 × 5 = 20 and 5 × 4 = 20



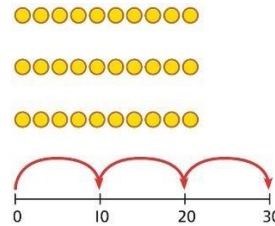
Learning $\times 2$, $\times 5$ and $\times 10$ table facts

Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.

3 groups of 10 ... 10, 20, 30
 $3 \times 10 = 30$

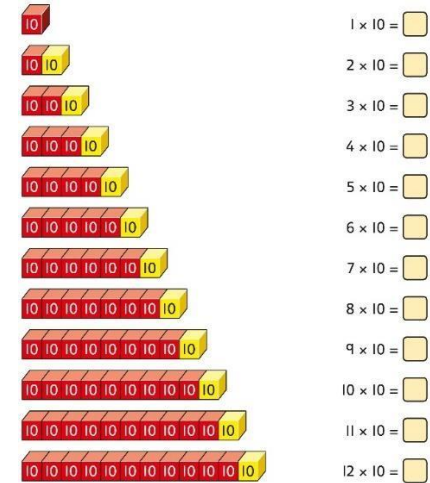


Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.



$10 + 10 + 10 = 30$
 $3 \times 10 = 30$

Understand how the times-tables increase and contain patterns.



$5 \times 10 = 50$
 $6 \times 10 = 60$

Year 2
Division

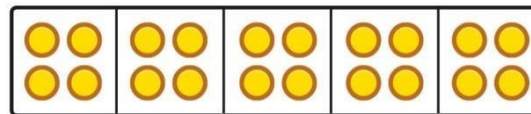
Sharing equally

Start with a whole and share into equal parts, one at a time.



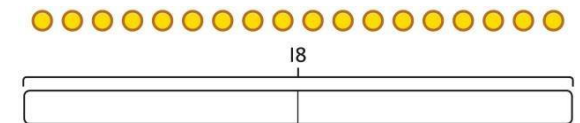
10 shared equally between 2. They get 5 each.

Represent the objects shared into equal parts using a bar model.



20 shared into 5 equal parts. There are 4 in each part.

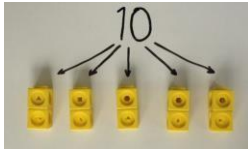
Use a bar model to support understanding of the division.



$18 \div 2 = 9$

Grouping equally

Understand how to make equal groups from a whole.



Understand the relationship between grouping and the division statements.

$$12 \div 3 = 4$$



$$12 \div 4 = 3$$



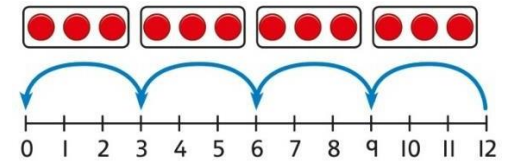
$$12 \div 6 = 2$$



$$12 \div 2 = 6$$



Understand how to relate division by grouping to repeated subtraction.



There are 4 groups now.

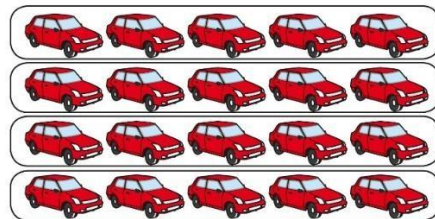
12 divided into groups of 3.

$$12 \div 3 = 4$$

There are 4 groups.

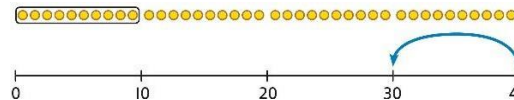
Using known times-tables to solve divisions

Understand the relationship between multiplication facts and division.



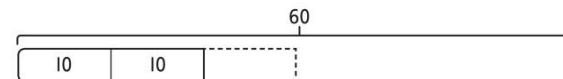
4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5.

Link equal grouping with repeated subtraction and known times-table facts to support division.



40 divided by 4 is 10.

Use a bar model to support understanding of the link between times-table knowledge and division.



Relate times-table knowledge directly to division.

$$\begin{aligned} 1 \times 10 &= 10 \\ 2 \times 10 &= 20 \\ \mathbf{3 \times 10} &= \mathbf{30} \\ 4 \times 10 &= 40 \\ 5 \times 10 &= 50 \\ 6 \times 10 &= 60 \\ 7 \times 10 &= 70 \\ 8 \times 10 &= 80 \end{aligned}$$

I used the 10 times-table to help me.
 $3 \times 10 = 30$.

I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.

$$3 \times 10 = 30 \quad \text{so} \quad 30 \div 10 = 3$$

LOWER KEY STAGE
2


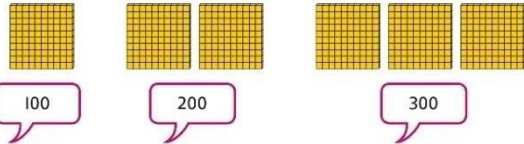
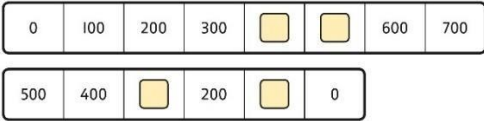
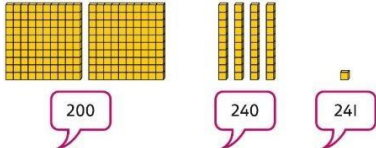
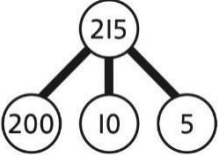
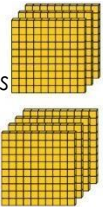
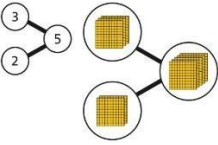
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

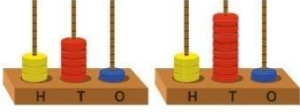
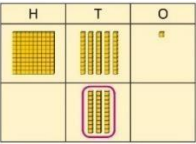
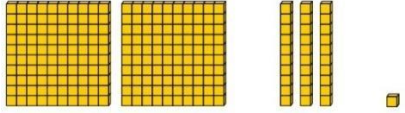
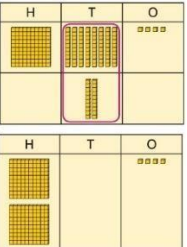
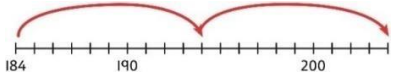

Addition and subtraction: In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply. In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

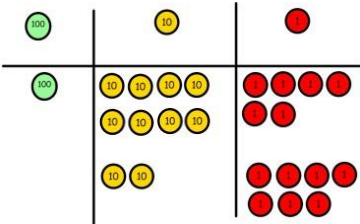
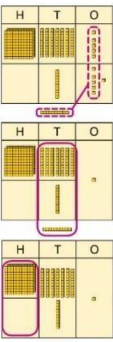
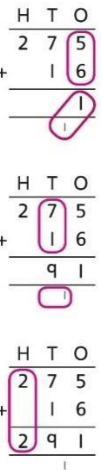
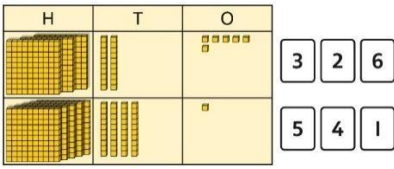
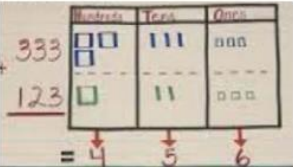
Multiplication and division: Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

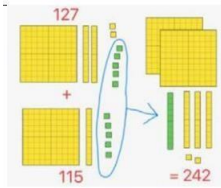
Fractions: Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount, and develop this with the aid of a bar model and other representations alongside. In Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1. Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

Year 3

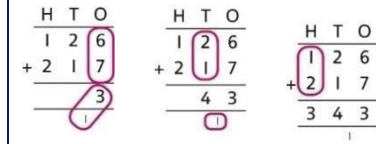
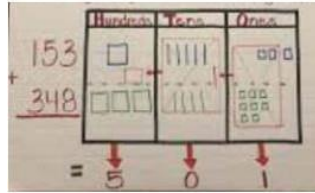
	Concrete	Pictorial	Abstract
Year 3 Addition			
Understanding 100s	<p>Understand the cardinality of 100, and the link with 10 tens.</p> <p>Use cubes to place into groups of 10 tens.</p> 	<p>Unitise 100 and count in steps of 100.</p> 	<p>Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0.</p> 
Understanding place value to 1,000	<p>Unitise 100s, 10s and 1s to build 3-digit numbers.</p>  <p>Use a place value grid to support the structure of numbers to 1,000.</p> <p>Place value counters are used alongside other equipment. Children should understand how each counter represents a different unitised amount.</p>	<p>Represent the parts of numbers to 1,000 using a part-whole model.</p>  <p>$215 = 200 + 10 + 5$</p>	<p>Recognise numbers to 1,000 represented on a numberline, including those between intervals.</p>
Adding 100s	<p>Use known facts and unitising to add multiples of 100.</p> <p>$3 + 4 = 7$ $3 \text{ hundreds} + 4 \text{ hundreds} = 7 \text{ hundreds}$ $300 + 400 = 700$</p> 	<p>Use known facts and unitising to add multiples of 100.</p> <p>$3 + 2 = 5$ $3 \text{ hundreds} + 2 \text{ hundreds} = 5 \text{ hundreds}$ $300 + 200 = 500$</p> 	<p>Use known facts and unitising to add multiples of 100.</p> <p>Represent the addition on a number line.</p> <p>Use a part-whole model to support unitising.</p> <p>$3 + 2 = 5$ $300 + 200 = 500$</p>

	 <p> $351 + 30 = ?$ $5 \text{ tens} + 3 \text{ tens} = 8 \text{ tens}$ $351 + 30 = 381$ </p>	 <p> $5 \text{ tens} + 3 \text{ tens} = 8 \text{ tens}$ $351 + 30 = 381$ </p>	<p><i>I know that $5 + 4 = 9$</i></p> <p>So, $50 + 40 = 90$ $753 + 40 = 793$</p>
<p>3-digit number + 10s, with exchange</p>	<p>Understand the exchange of 10 tens for 1 hundred.</p> 	<p>Add by exchanging 10 tens for 1 hundred.</p> <p>$184 + 20 = ?$</p>  <p>$184 + 20 = 204$</p>	<p>Understand how the addition relates to counting on in 10s across 100.</p>  <p>$184 + 20 = ?$</p> <p><i>I can count in 10s ... 194 ... 204</i></p> <p>$184 + 20 = 204$</p> <p>Use number bonds within 20 to support efficient calculations.</p> <p>$385 + 50$ <i>There are 8 tens and 5 tens. That is 13 tens.</i> $385 + 50 = 300 + 130 + 5$ $385 + 50 = 435$</p>
<p>3-digit number + 2-digit number</p>	<p>Use place value equipment to make and combine groups to model addition.</p> 	<p>Use a place value grid to organise thinking and adding of 1s, then 10s.</p>	<p>Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.</p>
<p>3-digit number + 2-digit number, exchange required</p>	<p>Use place value equipment to model addition and understand where exchange is required.</p> <p><i>Use place value counters to represent</i></p>	<p>Represent the required exchange on a place value grid.</p> <p>$275 + 16 = ?$</p>	<p>Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.</p>

	<p>$186 + 27$.</p> <p>Use this to decide if any exchange is required.</p> <p>There are 8 tens and 2 tens. That is 10 tens so I will exchange.</p> 	 <p>$275 + 16 = 291$</p> <p>Note: In this example, a mental method may be more efficient. The numbers for the example calculation have been chosen to allow children to visualise the concept and see how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.</p>	 <p>$275 + 16 = 291$</p>
<p>3-digit number + 3-digit number, no exchange</p>	<p>Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid.</p> <p>$326 + 541$ is represented as:</p> 	<p>Represent the place value grid with equipment to model the stages of column addition.</p>  <p>The calculation is shown alongside the model to expose the structure.</p>	<p>Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.</p> $\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$
<p>3-digit number + 3-digit number, exchange required</p>	<p>Use place value equipment to enact the exchange required.</p>	<p>Model the stages of column addition using place value equipment on a place value grid.</p>	<p>Use column addition, ensuring understanding of place value at every stage of the calculation.</p>



There are 13 ones.
I will exchange 10 ones for 1 ten.



$$126 + 217 = 343$$

Note: Children should also study examples where exchange is required in more than one column, for example $185 + 318 = ?$

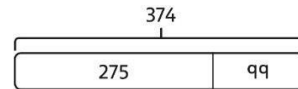
Representing addition problems, and selecting appropriate methods

Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps.

These representations will help them to select appropriate methods.

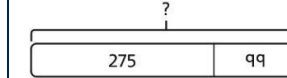
Children understand and create bar models to represent addition problems.

$$275 + 99 = ?$$



$$275 + 99 = 374$$

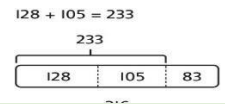
Use representations to support choices of appropriate methods.



I will add 100, then subtract 1 to find the solution.

$$128 + 105 + 83 = ?$$

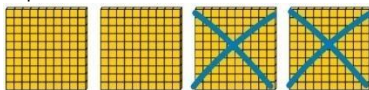
I need to add three numbers.



Year 3 Subtraction

Subtracting 100s

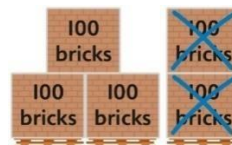
Use known facts and unitising to subtract multiples of 100. Expose the structure alongside pictorial representations.



$$4 - 2 = 2$$

$$400 - 200 = 200$$

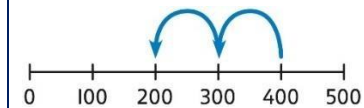
Use known facts and unitising to subtract multiples of 100.



$$5 - 2 = 3$$

$$500 - 200 = 300$$

Understand the link with counting back in 100s.



$$400 - 200 = 200$$

Use known facts and unitising as efficient and accurate methods.
I know that $7 - 4 = 3$. Therefore, I know that $700 - 400 = 300$.

3-digit number – 1s, no exchange

Use number bonds to subtract the 1s.

H	T	O
3	1	9

$$319 - 4 = ?$$

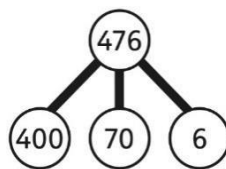
H	T	O
3	1	5

$$9 - 4 = 5$$

$$319 - 4 = 315$$

Use number bonds to subtract the 1s.

$$476 - 4 = ?$$



$$6 - 4 = 2$$

$$476 - 4 = 472$$

Understand the link with counting back using an number line.

Use known number bonds to calculate mentally.

3-digit number – 1s, exchange or bridging required

Understand why an exchange is necessary by exploring why 1 ten must be exchanged.

Use place value equipment.

H	T	O

$$151 - 6 = ?$$

Represent the required exchange on a place value grid.

H	T	O
0	00000	000000 00000
		000000

Calculate mentally by using known bonds.

$$151 - 6 = ?$$

$$151 - 1 - 5 = 145$$

3-digit number – 10s, no exchange

Subtract the 10s using known bonds.

$$381 - 10 = ?$$

8 tens with 1 removed is 7 tens.

$$381 - 10 = 371$$



Subtract the 10s using known bonds.

H	T	O

$$8 \text{ tens} - 1 \text{ ten} = 7 \text{ tens}$$

$$381 - 10 = 371$$

Use known bonds to subtract the 10s mentally.

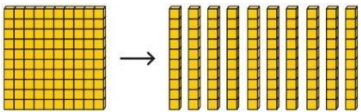
$$372 - 50 = ?$$

$$70 - 50 = 20$$

So, $372 - 50 = 322$



3-digit number -10s, exchange or bridging required

Use equipment to understand the exchange of 1 hundred for 10 tens.





Represent the exchange on a place value grid using equipment.

$210 - 20 = ?$

H	T	O
		

I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.

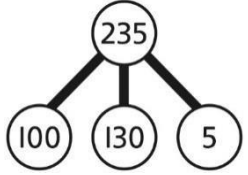
H	T	O
		

$210 - 20 = 190$

Understand the link with counting back on a numberline.

Use flexible partitioning to support the calculation.

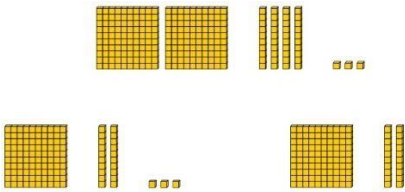
$235 - 60 = ?$



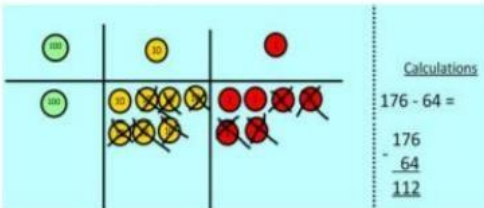
$235 = 100 + 130 + 5$
 $235 - 60 = 100 + 70 + 5$
 $= 175$

3-digit number - up to 3-digit number

Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away.



Represent the calculation on a place value grid.



Calculations

$$176 - 64 =$$

$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$$

Use column subtraction to calculate accurately and efficiently.

	H	T	O
	9	9	9
-	3	5	2
	<hr/>		
			7

	H	T	O
	9	9	9
-	3	5	2
	<hr/>		
			7

	H	T	O
	9	9	9
-	3	5	2
	<hr/>		
	6	4	7

3-digit number - up to 3-digit number, exchange required

Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.

Model the required exchange on a place value grid.

$175 - 38 = ?$

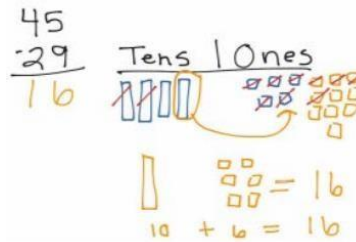
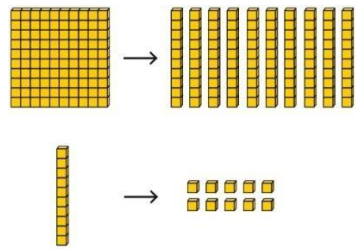
I need to subtract 8 ones, so I will exchange a ten for 10 ones.

Use column subtraction to work accurately and efficiently

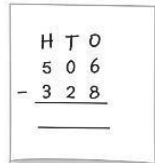
	H	T	O
	1	7	5
-		3	8
	<hr/>		
	1	3	7

$175 - 38 = 137$

175 - 38 = 137



If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly.



Children should also understand how to exchange in calculations where there is a zero in the 10s column.

Representing subtraction problems

Use bar models to represent subtractions.

'Find the difference' is represented as two bars for comparison.

Team A 454

Team B 128 \longleftrightarrow ?

Bar models can also be used to show that a part must be taken away from the whole.

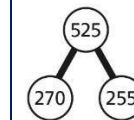
Children use alternative representations to check calculations and choose efficient methods.

Children use inverse operations to check additions and subtractions. The part-whole model supports understanding.

I have completed this subtraction.

$$525 - 270 = 255$$

I will check using addition.

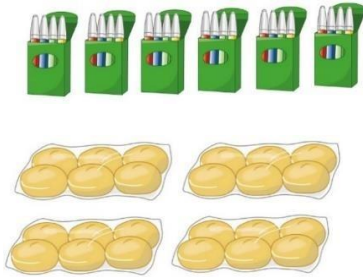


$$\begin{array}{r} \text{H T O} \\ 270 \\ + 255 \\ \hline 525 \\ \hline \end{array}$$

Year 3
Multiplication

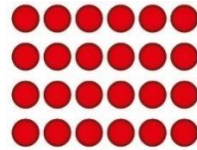
Using commutativity to support understanding of the times-tables

Understand how to use times-tables facts flexibly.



There are 6 groups of 4 pens.
There are 4 groups of 6 bread rolls.
I can use $6 \times 4 = 24$ to work out both totals.

Understand how times-table facts relate to commutativity.



$6 \times 4 = 24$
 $4 \times 6 = 24$

Understand how times-table facts relate to commutativity.

I need to work out 4 groups of 7.

I know that $7 \times 4 =$

28 so, I know that

4 groups of 7 =
28 and
7 groups of 4 = 28.

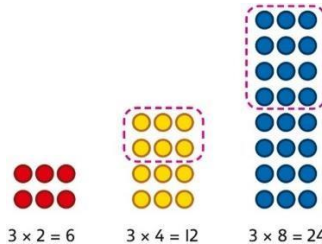
Understanding and using $\times 3$, $\times 2$, $\times 4$ and $\times 8$ tables.

Children learn the times-tables as 'groups of', but apply their knowledge of commutativity.



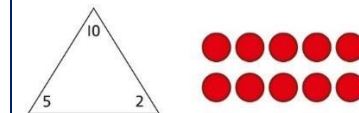
I can use the $\times 3$ table to work out how many keys. I can also use the $\times 3$ table to work out how many batteries.

Children understand how the $\times 2$, $\times 4$ and $\times 8$ tables are related through repeated doubling.



$3 \times 2 = 6$ $3 \times 4 = 12$ $3 \times 8 = 24$

Children understand the relationship between related multiplication and division facts in known times-tables.



$2 \times 5 = 10$
 $5 \times 2 = 10$
 $10 \div 5 = 2$
 $10 \div 2 = 5$

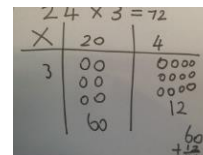
Multiplying a 2-digit number by a 1-digit number

Understand how to link partitioning a 2-digit number with multiplying.
Each person has 23 flowers.
Each person has 2 tens and 3 ones.



Use place value to support how partitioning is linked with multiplying by a 2-digit number.

$3 \times 24 = ?$
 $3 \times 4 = 12$
 $3 \times 20 = 60$
 $60 + 12 = 72$
 $3 \times 24 = 72$



Use addition to complete multiplications of 2-digit numbers by a 1-digit number.

$4 \times 13 = ?$
 $4 \times 3 = 12$ $4 \times 10 = 40$
 $12 + 40 = 52$ $4 \times 13 = 52$

There are 3 groups of 2 tens. There are 3 groups of 3 ones.
Use place value equipment to model the multiplication context alongside the representation.

x	T	U

There are 4 groups of 3 ones.
There are 4 groups of 1 tens.

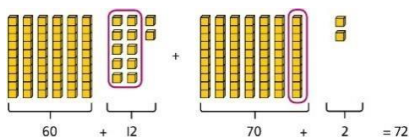
Multiplying a 2-digit number by a 1-digit number, expanded column method

Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications.

$$3 \times 24 = ?$$

$$3 \times 20 = 60$$

$$3 \times 4 = 12$$



$$3 \times 24 = 60 + 12$$

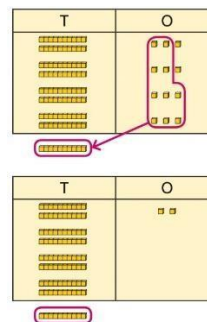
$$3 \times 24 = 70 + 2$$

$$3 \times 24 = 72$$

Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s.

$$4 \times 23 = ?$$

$$4 \times 23 = 92$$



Children may write calculations in expanded column form, but must understand the link with place value and exchange.

Children are encouraged to write the expanded parts of the calculation separately.

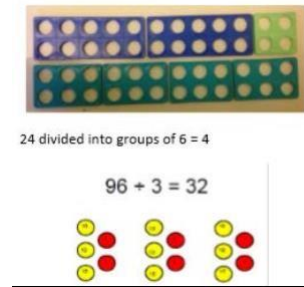
$$5 \times 28 = ?$$

$$\begin{array}{r} \text{T O} \\ \hline 28 \\ \times 5 \\ \hline 40 \quad 5 \times 8 \\ 100 \quad 5 \times 20 \\ \hline 140 \end{array}$$

Year 3
Division

Using times-tables knowledge to divide

Use knowledge of known times-tables to calculate divisions.

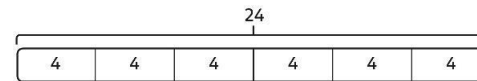


Use knowledge of known times-tables to calculate divisions.

I need to work out 30 shared between 5.

*I know that $6 \times 5 = 30$
so I know that $30 \div 5 = 6$.*

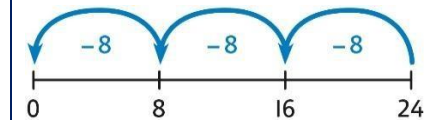
A bar model may represent the relationship between sharing and grouping.



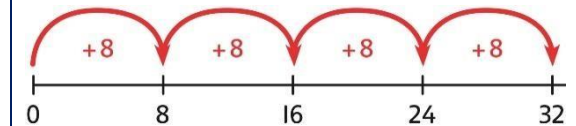
$24 \div 4 = 6$
 $24 \div 6 = 4$

Use knowledge of known times-tables to calculate divisions.

Children understand how division is related to both repeated subtraction and repeated addition.



$24 \div 8 = 3$



$32 \div 8 = 4$

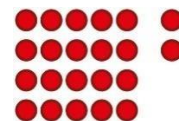
Understanding remainders

Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.



*There are 13 sticks in total.
There are 3 groups of 4, with 1 remainder.*

Use images to explain remainders.



$22 \div 5 = 4 \text{ remainder } 2$

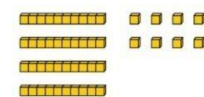
Understand that the remainder is what cannot be shared equally from a set.

$22 \div 5 = ?$

$3 \times 5 = 15$
 $4 \times 5 = 20$
 $5 \times 5 = 25 \dots \text{this is larger than } 22$
So, $22 \div 5 = 4 \text{ remainder } 2$

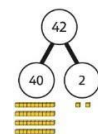
2-digit number divided by 1-digit number, no remainders

Children explore dividing 2-digit numbers by using place value equipment.



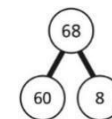
$48 \div 2 = ?$

Children explore which partitions support particular divisions.

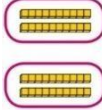

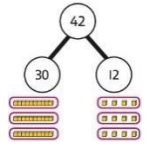
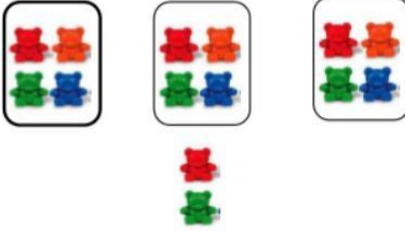
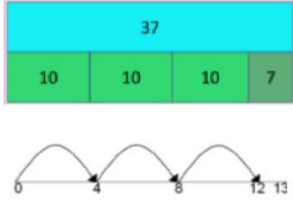
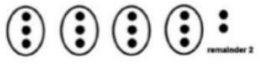


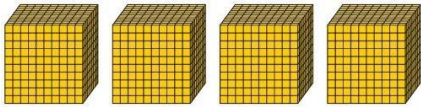

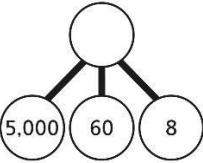


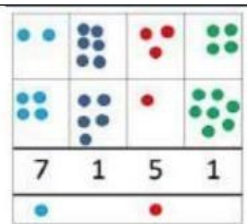
Children partition a number into 10s and 1s to divide where appropriate.

$68 \div 2 = ?$
 $60 \div 2 = 30$
 $8 \div 2 = 4$
 $30 + 4 = 34$



Children partition flexibly to divide where appropriate.

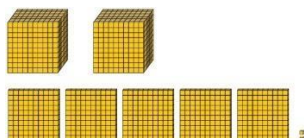
	<p>First divide the 10s.</p>  <p>Then divide the 1s.</p> 	<p>I need to partition 42 differently to divide by 3.</p>  <p>$42 = 30 + 12$</p> <p>$42 \div 3 = 14$</p>	<p>$42 \div 3 = ?$ $42 = 40 + 2$</p> <p>I need to partition 42 differently to divide by 3.</p> <p>$42 = 30 + 12$</p> <p>$30 \div 3 = 10$ $12 \div 3 = 4$</p> <p>$10 + 4 = 14$ $42 \div 3 = 14$</p>
<p>2-digit number divided by 1-digit number, with remainders</p>	<p>Use place value equipment to understand the concept of remainder.</p> 	<p>Use place value equipment to understand the concept of remainder in division.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p> 	<p>Partition to divide, understanding the remainder in context.</p> <p>67 children try to make 5 equal</p> <p>lines. $67 = 50 + 17$ $50 \div 5 = 10$</p> <p>$17 \div 5 = 3$ remainder 2 $67 \div 5 = 13$ remainder 2</p> <p>There are 13 children in each line and 2 children left out.</p>

	Concrete	Pictorial	Abstract																																																																																
Year 4 Addition																																																																																			
Understanding numbers to 10,000	<p>Use place value equipment to understand the place value of 4-digit numbers.</p>  <p>4 thousands equal 4,000.</p> <p>1 thousand is 10 hundreds.</p>  <p>$2,000 + 500 + 40 + 2 = 2,542$</p>	<p>Represent numbers using part-part-whole once children understand the relationship between 1,000s and 100s.</p>  <p>$5,000 + 60 + 8 = 5,068$</p>	<p>Understand partitioning of 4-digit numbers, including numbers with digits of 0.</p> <p>Understand and read 4-digit numbers on a numberline.</p> 																																																																																
Column addition with exchange	<p>Use place value equipment on a place value grid to organise thinking.</p>  <p>Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers.</p>	<p>Use place value equipment to model required exchanges.</p>  <p>Include examples that exchange in more than one column.</p>	<p>Use a column method to add, including exchanges.</p> <table style="display: inline-table; margin-right: 20px;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td>1</td><td>5</td><td>5</td><td>4</td></tr> <tr><td>+ 4</td><td>2</td><td>3</td><td>7</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td>9</td><td>1</td></tr> </table> <table style="display: inline-table; margin-right: 20px;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td>1</td><td>5</td><td>5</td><td>4</td></tr> <tr><td>+ 4</td><td>2</td><td>3</td><td>7</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td>7</td><td>9</td><td>1</td></tr> </table> <table style="display: inline-table; margin-right: 20px;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td>1</td><td>5</td><td>5</td><td>4</td></tr> <tr><td>+ 4</td><td>2</td><td>3</td><td>7</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td></td><td></td><td>9</td><td>1</td></tr> </table> <table style="display: inline-table;"> <tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr> <tr><td>1</td><td>5</td><td>5</td><td>4</td></tr> <tr><td>+ 4</td><td>2</td><td>3</td><td>7</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td>5</td><td>7</td><td>9</td><td>1</td></tr> </table> <p>Include examples that exchange in more than one column.</p>	Th	H	T	O	1	5	5	4	+ 4	2	3	7	<hr/>						9	1	Th	H	T	O	1	5	5	4	+ 4	2	3	7	<hr/>					7	9	1	Th	H	T	O	1	5	5	4	+ 4	2	3	7	<hr/>						9	1	Th	H	T	O	1	5	5	4	+ 4	2	3	7	<hr/>				5	7	9	1
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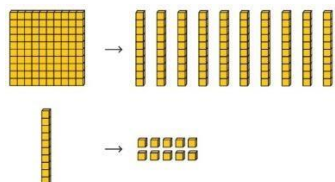
Column subtraction with exchange

Understand why one or more exchanges may be necessary.

$$2,502 - 243 = ?$$

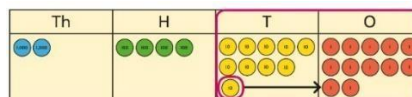
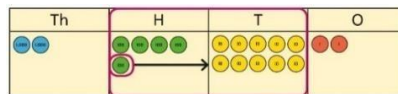


I need to exchange a 10 for some 1s, but there are not any 10s here.



Make exchanges across more than one column where there is a zero as a placeholder.

$$2,502 - 243 = ?$$



Make exchanges across more than one column where there is a zero as a placeholder.

$$2,502 - 243 = ?$$

Th	H	T	O
2	4 0	2	2
-	2	4	3

Th	H	T	O
2	4 9	0 12	2
-	2	4	3

Th	H	T	O
2	4 9	0 12	2
-	2	4	3
2	2	5	9

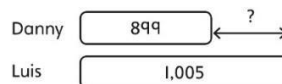
Representing subtractions and checking strategies

Use bar models to represent subtractions where a part needs to be calculated.



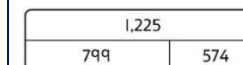
I can work out the total number of Yes votes using $5,762 - 2,899$.

Bar models can also represent 'find the difference' as a subtraction problem.



Use inverse operations to check subtractions.

I calculated $1,225 - 799 = 574$.
I will check by adding the parts.



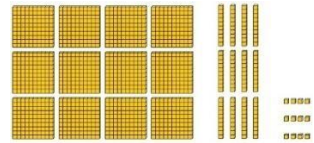
Th	H	T	O
7	9	9	
+	5	7	4
1	3	7	3

The parts do not add to make 1,225. I must have made a mistake.

Year 4
Multiplication

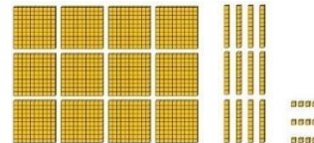
Multiplying by multiples of 10 and 100

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



3 groups of 4 ones is 12 ones.
3 groups of 4 tens is 12 tens.
3 groups of 4 hundreds is 12 hundreds.

Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.



$3 \times 4 = 12$
 $3 \times 40 = 120$
 $3 \times 400 = 1,200$

Use known facts and understanding of place value and commutativity to multiply mentally.

$$4 \times 7 = 28$$

$$4 \times 70 = 280$$

$$40 \times 7 = 280$$

$$4 \times 700 = 2,800$$

$$400 \times 7 = 2,800$$

Understanding times-tables up to 12×12

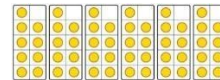
Understand the special cases of multiplying by 1 and 0.



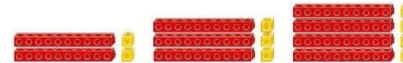
$$5 \times 1 = 5$$

$$5 \times 0 = 0$$

Represent the relationship between the $\times 9$ table and the $\times 10$ table.



Represent the $\times 11$ table and $\times 12$ tables in relation to the $\times 10$ table.



$$2 \times 11 = 20 + 2$$

$$3 \times 11 = 30 + 3$$

$$4 \times 11 = 40 + 4$$



$$4 \times 12 = 40 + 8$$

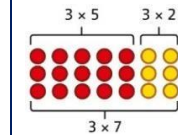
Understand how times-tables relate to counting patterns.

Understand links between the $\times 3$ table, $\times 6$ table and $\times 9$ table
 5×6 is double 5×3

$\times 5$ table and $\times 6$ table
I know that $7 \times 5 = 35$
so I know that $7 \times 6 = 35 + 7$.

$\times 5$ table and $\times 7$ table

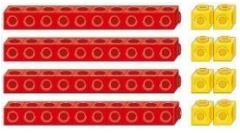
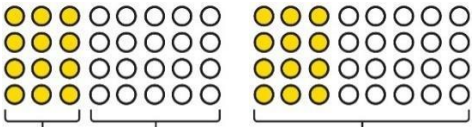
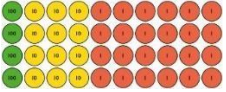
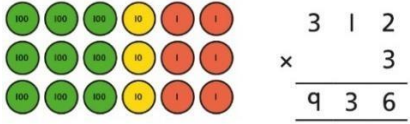
$$3 \times 7 = 3 \times 5 + 3 \times 2$$



$\times 9$ table and $\times 10$ table

$$6 \times 10 = 60$$

$$6 \times 9 = 60 - 6$$

<p>Understanding and using partitioning in multiplication</p>	<p>Make multiplications by partitioning.</p> <p>4×12 is 4 groups of 10 and 4 groups of 2.</p>  <p>$4 \times 12 = 40 + 8$</p>	<p>Understand how multiplication and partitioning are related through addition.</p>  <p>$4 \times 3 = 12$ $4 \times 5 = 20$ $12 + 20 = 32$</p> <p>$4 \times 8 = 32$</p>	<p>Use partitioning to multiply 2-digit numbers by a single digit.</p> <p>$18 \times 6 = ?$</p> $18 \times 6 = 10 \times 6 + 8 \times 6$ $= 60 + 48$ $= 108$
<p>Column multiplication for 2- and 3-digit numbers multiplied by a single digit</p>	<p>Use place value equipment to make multiplications.</p> <p>Make 4×136 using equipment.</p>  <p>I can work out how many 1s, 10s and 100s.</p> <p>There are 4×6 ones... 24 ones There are 4×3 tens ... 12 tens There are 4×1 hundreds ... 4 hundreds</p> <p>$24 + 120 + 400 = 544$</p>	<p>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.</p>  $\begin{array}{r} 312 \\ \times 4 \\ \hline 1248 \end{array}$	<p>Use the formal column method for up to 3-digit numbers multiplied by a single digit.</p> $\begin{array}{r} 312 \\ \times 3 \\ \hline 936 \end{array}$ <p>Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.</p> $\begin{array}{r} 23 \\ \times 5 \\ \hline 115 \end{array}$ $\begin{array}{r} 23 \\ \times 5 \\ \hline 115 \end{array}$
<p>Multiplying more than two numbers</p>	<p>Represent situations by multiplying three numbers together.</p>	<p>Understand that commutativity can be used to multiply in different orders.</p>	<p>Use knowledge of factors to simplify some multiplications.</p> <p>$24 \times 5 = 12 \times 2 \times 5$</p>



Each sheet has 2×5 stickers. There are 3 sheets.

There are $5 \times 2 \times 3$ stickers in total.

$$\begin{array}{l} 5 \times 2 \times 3 = 30 \\ \underbrace{\hspace{1.5cm}} \\ 10 \times 3 = 30 \end{array}$$



$$\begin{array}{l} 2 \times 6 \times 10 = 120 \\ 12 \times 10 = 120 \end{array}$$

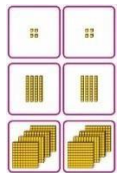
$$\begin{array}{l} 10 \times 6 \times 2 = 120 \\ 60 \times 2 = 120 \end{array}$$

$$\begin{array}{l} 12 \times 2 \times 5 = \\ \underbrace{\hspace{1.5cm}} \\ 12 \times 10 = 120 \\ \text{So, } 24 \times 5 = 120 \end{array}$$

Year 4
Division

Dividing multiples of 10 and 100 by a single digit

Use place value equipment to understand how to use unitising to divide.

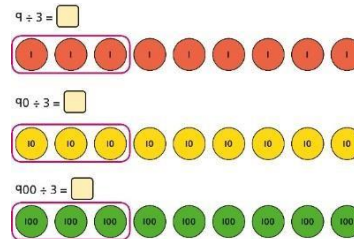


8 ones divided into 2 equal groups 4 ones in each group

8 tens divided into 2 equal groups 4 tens in each group

8 hundreds divided into 2 equal groups 4 hundreds in each group

Represent divisions using place value equipment.



$$9 \div 3 = 3$$

9 tens divided by 3 is 3 tens.
9 hundreds divided by 3 is 3 hundreds.

Use known facts to divide 10s and 100s by a single digit.

$$15 \div 3 = 5$$

$$150 \div 3 = 50$$

$$1500 \div 3 = 500$$

Dividing 2-digit and 3-digit numbers by a single digit by

Partition into 10s and 1s to divide where appropriate.

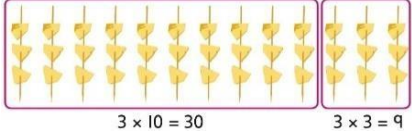
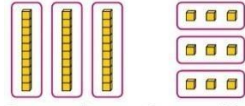
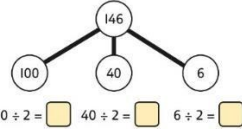
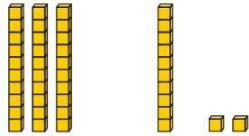
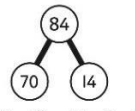
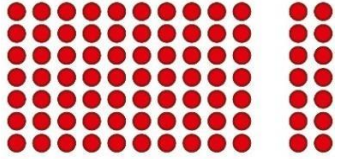
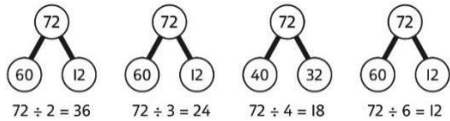
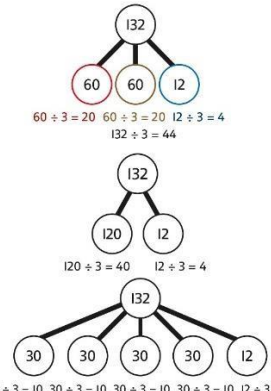
$$39 \div 3 = ?$$

Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate.

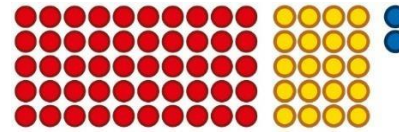
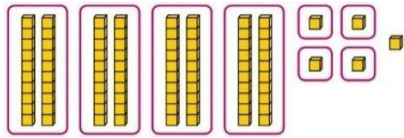
$$39 \div 3 = ?$$

Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate.

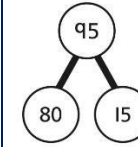
$$142 \div 2 = ?$$

<p>partitioning into 100s, 10s and 1s</p>	 <p>$3 \times 10 = 30$ $3 \times 3 = 9$</p> <p>$39 = 30 + 9$</p> <p>$30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$</p>	 <p>3 groups of 1 ten 3 groups of 3 ones</p> <p>$39 = 30 + 9$</p> <p>$30 \div 3 = 10$ $9 \div 3 = 3$ $39 \div 3 = 13$</p>	 <p>$100 \div 2 = 50$ $40 \div 2 = 20$ $6 \div 2 = 3$ $50 + 20 + 3 = 73$ $142 \div 2 = 73$</p>
<p>Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning</p>	<p>Use place value equipment to explore why different partitions are needed.</p> <p>$42 \div 3 = ?$</p> <p><i>I will split it into 30 and 12, so that I can divide by 3 more easily.</i></p> 	<p>Represent how to partition flexibly where needed.</p> <p>$84 \div 7 = ?$</p> <p><i>I will partition into 70 and 14 because I am dividing by 7.</i></p>  <p>$70 \div 7 = 10$ $14 \div 7 = 2$</p> <p>$84 \div 7 = 12$</p> 	<p>Make decisions about appropriate partitioning based on the division required.</p>  <p>$72 \div 2 = 36$ $72 \div 3 = 24$ $72 \div 4 = 18$ $72 \div 6 = 12$</p> <p>Understand that different partitions can be used to complete the same division.</p>  <p>$60 \div 3 = 20$ $60 \div 3 = 20$ $12 \div 3 = 4$ $132 \div 3 = 44$</p> <p>$120 \div 3 = 40$ $12 \div 3 = 4$</p> <p>$30 \div 3 = 10$ $30 \div 3 = 10$ $30 \div 3 = 10$ $30 \div 3 = 10$ $12 \div 3 = 4$</p>
<p>Understanding remainders</p>	<p>Use place value equipment to find remainders.</p> <p><i>85 shared into 4 equal groups</i></p>	<p>Represent the remainder as the part that cannot be shared equally.</p>	<p>Understand how partitioning can reveal remainders of divisions.</p>

There are 24, and 1 that cannot be shared.



$$72 \div 5 = 14 \text{ remainder } 2$$



$$80 \div 4 = 20$$

$$12 \div 4 = 3$$

$$95 \div 4 = 23 \text{ remainder } 3$$

UPPER KEY STAGE
2

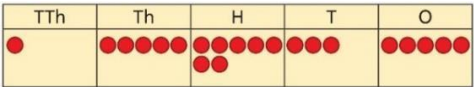
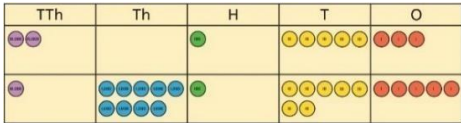
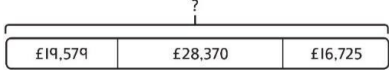
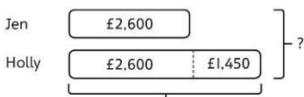
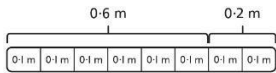
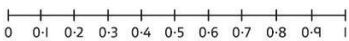
In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

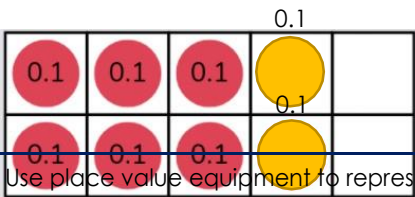
Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined.

Fractions: Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

Year 5

	Concrete	Pictorial	Abstract
Year 5 Addition			
Column addition with whole numbers	<p>Use place value equipment to represent additions.</p> <p>Add a row of counters onto the place value grid to show $15,735 + 4,012$.</p> 	<p>Represent additions, using place value equipment on a place value grid alongside written methods.</p>  <p>I need to exchange 10 tens for a 100.</p> $\begin{array}{r} \text{TTh Th H T O} \\ 2 \ 0 \ 1 \ 5 \ 3 \\ + 1 \ 4 \ 0 \ 1 \ 2 \\ \hline 3 \ 4 \ 1 \ 6 \ 5 \end{array}$	<p>Use column addition, including exchanges.</p> $\begin{array}{r} \text{TTh Th H T O} \\ 1 \ 9 \ 1 \ 7 \ 5 \\ + 1 \ 8 \ 4 \ 1 \ 7 \\ \hline 3 \ 7 \ 5 \ 8 \ 2 \\ \hline \end{array}$
Representing additions		<p>Bar models represent addition of two or more numbers in the context of problem solving.</p>  <p>Jen: £2,600 Holly: £2,600 and £1,450</p>  <p>£4,050</p> $\begin{array}{r} \text{Th H T O} \\ 2 \ 6 \ 0 \ 0 \\ + 1 \ 4 \ 5 \ 0 \\ \hline 4 \ 0 \ 5 \ 0 \end{array} \quad \begin{array}{r} \text{Th H T O} \\ 2 \ 6 \ 0 \ 0 \\ + 4 \ 0 \ 5 \ 0 \\ \hline 6 \ 6 \ 5 \ 0 \end{array}$	<p>Use approximation to check whether answers are reasonable.</p> $\begin{array}{r} \text{TTh Th H T O} \\ 2 \ 3 \ 4 \ 0 \ 5 \\ + 7 \ 8 \ 9 \ 2 \\ \hline 2 \ 0 \ 2 \ 9 \ 7 \end{array} \quad \begin{array}{r} \text{TTh Th H T O} \\ 2 \ 3 \ 4 \ 0 \ 5 \\ + 7 \ 8 \ 9 \ 2 \\ \hline 3 \ 1 \ 2 \ 9 \ 7 \\ \hline \end{array}$ <p>I will use $23,000 + 8,000$ to check.</p>
Adding tenths	<p>Link measure with addition of decimals.</p> <p>Two lengths of fencing are 0.6 m and 0.2 m. How long are they when added together?</p>	<p>Use a bar model with a number line to add tenths.</p>  	<p>Understand the link with adding fractions.</p> $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$



$$0.6 + 0.2 = 0.8$$

$$6 \text{ tenths} + 2 \text{ tenths} = 8 \text{ tenths}$$

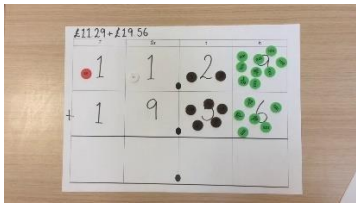
$$6 \text{ tenths} + 2 \text{ tenths} = 8$$

$$\text{tenths } 0.6 + 0.2 = 0.8$$

Adding decimals using column addition

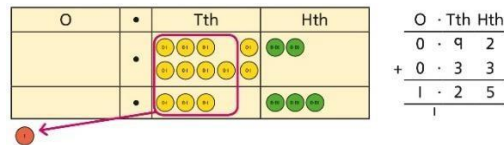
Use place value equipment to represent additions.

Show $0.23 + 0.45$ using place value counters.



Use place value equipment on a place value grid to represent additions.

Represent exchange where necessary.



Include examples where the numbers of decimal places are different.



Add using a column method, ensuring that children understand the link with place value.

$$\begin{array}{r} 0.23 \\ + 0.45 \\ \hline 0.68 \end{array}$$

Include exchange where required, alongside an understanding of place value.

$$\begin{array}{r} 0.92 \\ + 0.33 \\ \hline 1.25 \end{array}$$

Include additions where the numbers of decimal places are different.

$$3.4 + 0.65 = ?$$

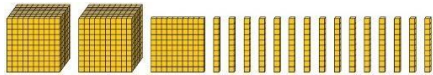
$$\begin{array}{r} 3.40 \\ + 0.65 \\ \hline \end{array}$$

Year 5
Subtraction

Column subtraction with whole numbers

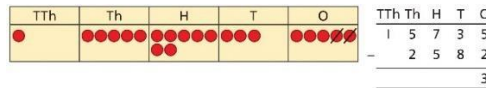
Use place value equipment to understand where exchanges are required.

$$2,250 - 1,070$$

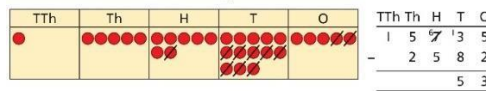


Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.

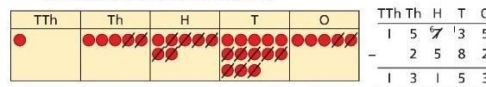
$$15,735 - 2,582 = 13,153$$



Now subtract the 10s. Exchange 1 hundred for 10 tens.



Subtract the 100s, 1,000s and 10,000s.



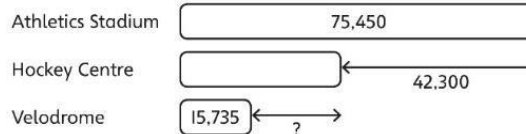
Use column subtraction methods with exchange where required. Drawing attention to accuracy and speed.

TTh	Th	H	T	O
5	12	10	9	7
-	1	8	5	3
	4	3	5	6
			3	

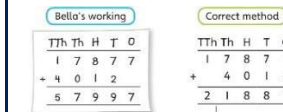
$$62,097 - 18,534 = 43,563$$

Checking strategies and representing subtractions

Bar models represent subtractions in problem contexts, including 'find the difference'.



Children can explain the mistake made when the columns have not been ordered correctly.



Use approximation to check calculations.

I calculated 18,000 + 4,000 mentally to check my subtraction.

Choosing efficient methods

To subtract two large numbers that are close, children find the difference by counting on.

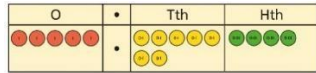


Use addition to check subtractions. *I calculated 7,546 - 2,355 = 5,191. I will check using the inverse.*

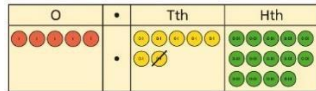
Subtracting decimals

Use a place value counters to represent the stages of column subtraction, including exchanges where required.

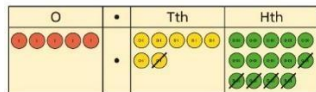
$$5.74 - 2.25 = ?$$



Exchange 1 tenth for 10 hundredths.



Now subtract the 5 hundredths.



Now subtract the 2 tenths, then the 2 ones.



Use a place value grid to represent the stages of column subtraction, including exchanges where required.

Explore complements to a whole number by working in the context of length.

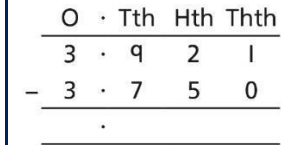


$$1 \text{ m} - \square \text{ m} = \square \text{ m}$$

$$1 - 0.49 = ?$$

Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.

$$3.921 - 3.75 = ?$$



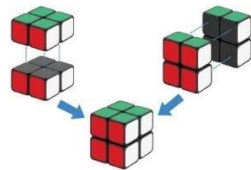
Year 5 Multiplication

Understanding factors

Use cubes or counters to explore the meaning of 'square numbers'.

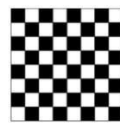
25 is a square number because it is made from 5 rows of 5.

Use cubes to explore cube numbers.



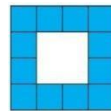
8 is a cube number.

Use images to explore examples and non-examples of square numbers.



$$8 \times 8 = 64$$

$$8^2 = 64$$

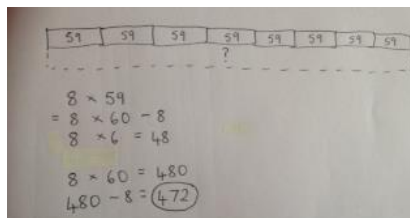


12 is not a square number, because you cannot multiply a whole number by itself to make 12.

Understand the pattern of square numbers in the multiplication tables.

Use a multiplication grid to circle each square number. Can children spot a pattern?

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



For children who are not secure with their knowledge of place value, they will use the expanded method:

$$\begin{array}{r} 2741 \\ \times 6 \\ \hline 6 \quad 6 \times 1 = 6 \text{ (ones x ones)} \\ 240 \quad 6 \times 40 = 240 \text{ (ones x tens)} \\ 4200 \quad 6 \times 700 = 4200 \text{ (ones x hundreds)} \\ 12000 \quad 6 \times 2000 = 12000 \text{ (ones x thousands)} \\ \hline 16446 \end{array}$$

Children should begin by writing down \times calculations they have done so mistakes can be easily identified. When accuracy improves these can be left out.

Multiplying up to 4-digits by 2-digits

Manipulatives may still be used with the corresponding long multiplication modelled alongside.

Model	Calculation

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

Use a column multiplication, including any required exchanges.

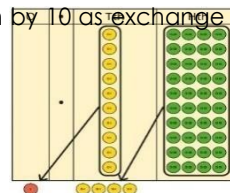
$$\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 240 \\ \hline 384 \end{array}$$

Multiplying decimals by 10, 100 and 1,000

Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.

Represent multiplication by 10 as exchange on a place value grid.

$$0.14 \times 10 = 1.4$$



Understand how this exchange is represented on a place value chart.

Th	H	T	O	•	Tth
			2	•	5
		2	5	•	
	2	5	0	•	
2	5	0	0	•	

$2.5 \times 10 = 25$
 $2.5 \times 100 = 250$
 $2.5 \times 1,000 = 2,500$

Year 5
Division

Understanding factors and prime numbers

Use equipment to explore the factors of a given number.



$$24 \div 3 = 8$$

$$24 \div 8 = 3$$

8 and 3 are factors of 24 because they divide 24 exactly.

$$24 \div 5 = 4 \text{ remainder } 4.$$



5 is not a factor of 24 because there is a remainder.

Understand that prime numbers are numbers with exactly two factors.

$$13 \div 1 = 13$$

$$13 \div 2 = 6 \text{ r } 1$$

$$13 \div 4 = 4 \text{ r } 1$$

1 and 13 are the only factors of 13. 13 is a prime number.

Understand how to recognise prime and composite numbers.

I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.

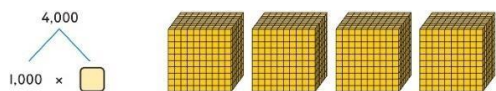
I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.

I know that 1 is not a prime number, as it has only 1 factor.

Dividing whole numbers by 10, 100 and 1,000

Use place value equipment to support unitising for division.

$$4,000 \div 1,000$$



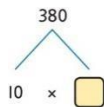
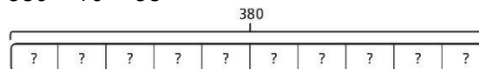
4,000 is 4 thousands.

$$4 \times 1,000 = 4,000$$

$$\text{So, } 4,000 \div 1,000 = 4$$

Use a bar model to support dividing by unitising.

$$380 \div 10 = 38$$



380 is 38 tens.

$$38 \times 10 = 380$$

$$10 \times 38 = 380$$

$$\text{So, } 380 \div 10 = 38$$

Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.

Th	H	T	O
3	2	0	0

$$3,200 \div 100 = ?$$

3,200 is 3 thousands and 2 hundreds.

$$200 \div 100 = 2$$

$$3,000 \div 100 = 30$$

$$3,200 \div 100 = 32$$

So, the digits will move two places to the right.

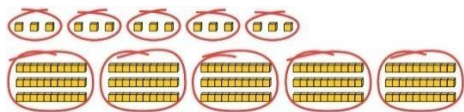
Dividing by multiples of 10, 100 and 1,000

Use place value equipment to represent known facts and unitising.

Represent related facts with place value equipment when dividing by unitising.

Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.

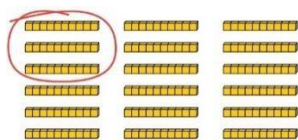
$$3,000 \div 5 = 600$$



15 ones put into groups of 3 ones. There are 5 groups. $15 \div 3 = 5$

15 tens put into groups of 3 tens. There are 5 groups.

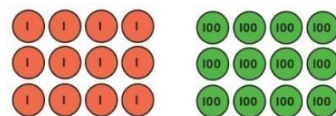
$$150 \div 30 = 5$$



180 is 18 tens.

18 tens divided into groups of 3 tens. There are 6 groups.

$$180 \div 30 = 6$$



12 ones divided into groups of 4. There are 3 groups.

12 hundreds divided into groups of 4 hundreds. There are 3 groups.

$$1200 \div 400 = 3$$

$$3,000 \div 50 = 60$$

$$3,000 \div 500 = 6$$

$$5 \times 600 = 3,000$$

$$50 \times 60 = 3,000$$

$$500 \times 6 = 3,000$$

Dividing up to four digits by a single digit using short division

Explore grouping using place value equipment.

$$268 \div 2 = ?$$

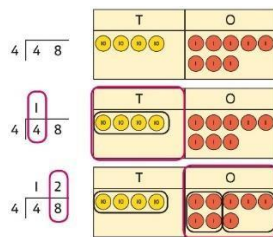
There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones.

$$264 \div 2 = 134$$

Use place value equipment on a place value grid alongside short division.

The model uses grouping.

A sharing model can also be used, although the model would need adapting.



Use short division for up to 4-digit numbers divided by a single digit.

$$\begin{array}{r} 0556 \\ 7 \overline{) 3892} \end{array}$$

$$3,892 \div 7 = 556$$

Use multiplication to check.

$$556 \times 7 = ?$$

$$6 \times 7 = 42$$

Lay out the problem as a short division.

There is 1 group of 4 in 4 tens.
There are 2 groups of 4 in 8 ones.

Work with divisions that require exchange.

4 $\overline{) 92}$

First, lay out the problem.

T	O
90	20

How many groups of 4 go into 9 tens?
2 groups of 4 tens with 1 ten left over.

2 $\overline{) 92}$

T	O
20	20

Exchange the 1 ten left over for 10 ones.
We now have 12 ones.

2 $\overline{) 92}$

T	O
20	12

How many groups of 4 go into 12 ones?
3 groups of 4 ones.

2 $\overline{) 92}$

T	O
23	2

$$50 \times 7 = 350$$

$$500 \times 7 = 3500$$

$$3,500 + 350 + 42 = 3,892$$

Understanding remainders

Understand remainders using concrete versions of a problem.

80 cakes divided into trays of 6.



80 cakes in total. They make 13 groups of 6, with 2 remaining.

Use short division and understand remainders as the last remaining 1s.

6 $\overline{) 80}$

T	O
80	

Lay out the problem as short division.

1 $\overline{) 80}$

T	O
10	20

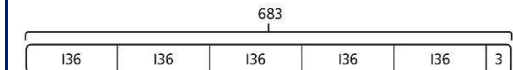
How many groups of 6 go into 8 tens?
There is 1 group of 6 tens.
There are 2 tens remaining.

1 $\overline{) 80}$

T	O
13	20

How many groups of 6 go into 20 ones?
There are 3 groups of 6 ones.
There are 2 ones remaining.

In problem solving contexts, represent divisions including remainders with a bar model.



$$683 = 136 \times 5 + 3$$

$$683 \div 5 = 136 \text{ r } 3$$

Dividing decimals by 10, 100 and 1,000

Understand division by 10 using exchange.

2 ones are 20 tenths.

20 tenths divided by 10 is 2 tenths.

Represent division using exchange on a place value grid.

1.5 is 1 one and 5 tenths.
 This is equivalent to 10 tenths and 50 hundredths.
 10 tenths divided by 10 is 1 tenth.
 50 hundredths divided by 10 is 5 hundredths.
 1.5 divided by 10 is 1 tenth and 5 hundredths. $1.5 \div 10 = 0.15$

Understand the movement of digits on a place value grid.

$0.85 \div 10 = 0.085$

$8.5 \div 100 = 0.085$

Understanding the relationship between fractions and division

Use sharing to explore the link between fractions and division.

1 whole shared between 3 people. Each person receives one-third.

Use a bar model and other fraction representations to show the link between fractions and division.

$1 \div 3 = \frac{1}{3}$

Use the link between division and fractions to calculate divisions.

$$5 \div 4 = \frac{5}{4} = 1 \frac{1}{4}$$

$$11 \div 4 = \frac{11}{4} = 2 \frac{3}{4}$$

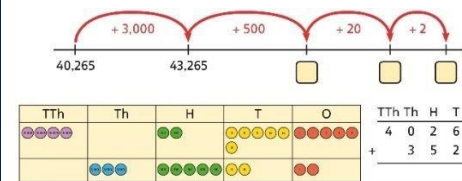
Year 6 Addition

Comparing and selecting efficient methods

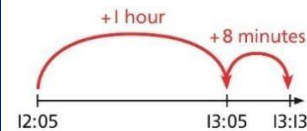
Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.



Discuss similarities and differences between methods, and choose efficient methods based on the specific calculation. Compare written and mental methods alongside place value representations.

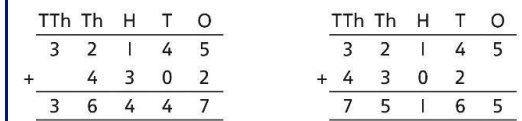


Use bar model and number line representations to model addition in problem-solving and measure contexts.



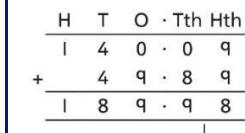
Use column addition where mental methods are not efficient. Recognise common errors with column addition.

$32,145 + 4,302 = ?$



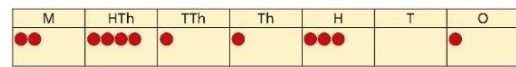
Which method has been completed accurately? What mistake has been made?

Column methods are also used for decimal additions where mental methods are not efficient.



Selecting mental methods for larger numbers where appropriate

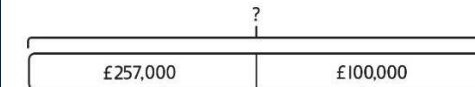
Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.



$2,411,301 + 500,000 = ?$

Use a bar model to support thinking in addition problems.

$257,000 + 99,000 = ?$



Use place value and unitising to support mental calculations with larger numbers.

$195,000 + 6,000 = ?$

$195 + 5 + 1 = 201$

195 thousands + 6 thousands = 201 thousands
So, $195,000 + 6,000 = 201,000$

This would be 5 more counters in the HTh place. So, the total is 2,911,301.

$$2,411,301 + 500,000 = 2,911,301$$

I added 100 thousands then subtracted 1 thousand.

$$257 \text{ thousands} + 100 \text{ thousands} = 357 \text{ thousands}$$

$$257,000 + 100,000 = 357,000$$

$$357,000 - 1,000 = 356,000$$

So, $257,000 + 99,000 = 356,000$

Understanding order of operations in calculations

Use equipment to model different interpretations of a calculation with more than one operation. Explore different results to draw attention to the structure.

$3 \times 5 - 2 = ?$

$$\begin{array}{r} 3 \times 5 - 2 \\ \downarrow \downarrow \\ 3 \times 3 = 9 \end{array}$$

$$\begin{array}{r} 3 \times 5 - 2 \\ \downarrow \downarrow \\ 15 - 2 = 13 \end{array}$$

Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations.

This can be written as: $16 \times 4 + 16 \times 6$

$$\boxed{16 \times 4} + \boxed{16 \times 6}$$

$$64 + 96 = 160$$

Understand the correct order of operations in calculations without brackets.

Understand how brackets affect the order of operations in a calculation.

$$4 + 6 \times 16$$

$$4 + 96 = 100$$

$$(4 + 6) \times 16$$

$$10 \times 16 = 160$$

Year 6 Subtraction

Comparing and selecting efficient methods

Use counters on a place value grid to represent subtractions of larger numbers.

Th	H	T	O
●●	●●●●●	●●●●●	●●●●●
	●	●●	●●●●

Compare subtraction methods alongside place value representations.

Th	H	T	O
●●	●●●●●	●●●●●	●●●●●
	●	●●	●●●●

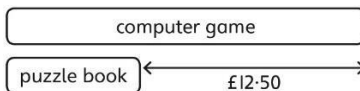
Compare and select methods. Use column subtraction when mental methods are not efficient.

Th	H	T	O
1	5	5	8
-	1	5	8
<hr/>			
	3	9	4

Use column subtraction for decimal problems, including in the context of measure.

	Th	H	T	O
	2	6	7	9
-		5	3	4
	2	1	4	5

Use a bar model to represent calculations, including 'find the difference' with two bars as comparison.

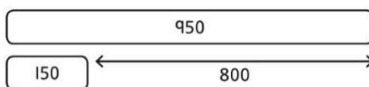


	H	T	O	·	Tth	Hth
	3	0	9	·	6	0
-		2	0	·	6	0
	1	0	3	·	2	0

Subtracting mentally with larger numbers

Use a bar model to show how unitising can support mental calculations.

$950,000 - 150,000$
That is 950 thousands - 150 thousands



So, the difference is 800 thousands.
 $950,000 - 150,000 = 800,000$

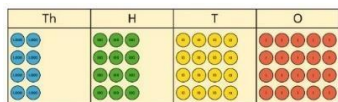
Subtract efficiently from powers of 10.

$10,000 - 500 = ?$

Year 6
Multiplication

Multiplying up to a 4-digit number by a single digit or 2-digit number.

Use equipment to explore multiplications alongside the written calculation.

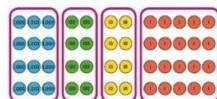


4 groups of 2,345

This is a multiplication:

$4 \times 2,345$
 $2,345 \times 4$

Use place value representations alongside the written method methods where necessary.



$4 \times 3,000 + 4 \times 200 + 4 \times 20 + 4 \times 5$
 $12,000 + 800 + 80 + 20 = 12,900$

Understand short multiplication.

$$\begin{array}{r} 3225 \\ \times \quad 4 \\ \hline 12900 \\ \quad 12 \\ \hline \end{array}$$

Multiplying up to a 4-digit number by a 2-digit number

Use an expanded form to expose the structure where necessary.

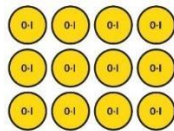
$$\begin{array}{r}
 1\ 2\ 3\ 5 \\
 \times \quad 2\ 1 \\
 \hline
 5 \quad 1 \times 5 \\
 3\ 0 \quad 1 \times 30 \\
 2\ 0\ 0 \quad 1 \times 200 \\
 1\ 0\ 0\ 0 \quad 1 \times 1,000 \\
 1\ 0\ 0 \quad 20 \times 5 \\
 6\ 0\ 0 \quad 20 \times 30 \\
 4\ 0\ 0\ 0 \quad 20 \times 200 \\
 2\ 0\ 0\ 0\ 0 \quad 20 \times 1,000 \\
 \hline
 2\ 5\ 9\ 3\ 5 \quad 21 \times 1,235
 \end{array}$$

Use compact column multiplication with understanding of place value at all stages.

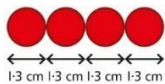
$$\begin{array}{r}
 1\ 2\ 3\ 5 \\
 \times \quad \quad 2\ 1 \\
 \hline
 1\ 2\ 3\ 5 \\
 2\ 4\ 7\ 0\ 0 \\
 \hline
 2\ 5\ 9\ 3\ 5
 \end{array}$$

Multiplying decimals

Explore decimal multiplications using place value equipment and in the context of measures.



3 groups of 4 tenths is 12 tenths.
4 groups of 3 tenths is 12 tenths.



$$\begin{aligned}
 4 \times 1\text{ cm} &= 4\text{ cm} \\
 4 \times 0.3\text{ cm} &= 1.2\text{ cm} \\
 4 \times 1.3 &= 4 + 1.2 = 5.2\text{ cm}
 \end{aligned}$$

Represent calculations on a place value grid.

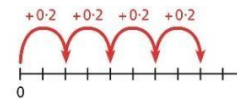
$$3 \times 3 = 9$$

$$3 \times 0.3 = 0.9$$

T	O	.	Tth
		.	● ● ●
		.	● ● ●
		.	● ● ●

Understand the link between multiplying decimals and repeated addition.

T	O	.	Tth
		.	● ● ●
		.	● ● ●



Use known facts to multiply decimals.

$$\begin{aligned}
 4 \times 3 &= 12 \\
 4 \times 0.3 &= 1.2 \\
 4 \times 0.03 &= 0.12
 \end{aligned}$$

$$\begin{aligned}
 20 \times 5 &= 100 \\
 20 \times 0.5 &= 10 \\
 20 \times 0.05 &= 1
 \end{aligned}$$

Find families of facts from a known multiplication.

I know that $18 \times 4 = 72$.

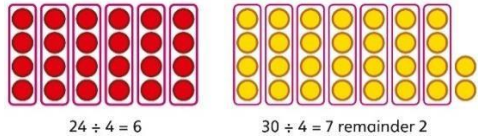
This can help me work

$$\begin{aligned}
 \text{out: } 1.8 \times 4 &= ? \\
 18 \times 0.4 &= ? \\
 180 \times 0.4 &= ? \\
 18 \times 0.04 &= ?
 \end{aligned}$$

Year 6
Division

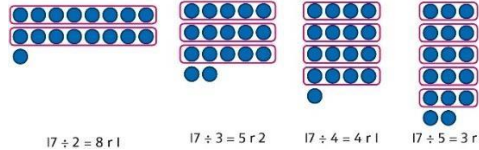
Understanding factors

Use equipment to explore different factors of a number.



4 is a factor of 24 but is not a factor of 30.

Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.

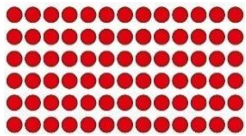


Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Dividing by a single digit

Use equipment to make groups from a total.



There are 78 in total.
There are 6 groups of 13.
There are 13 groups of 6.

H	T	O
●	●●	●●

How many groups of 6 are in 100? $6 \overline{) 100} \begin{matrix} 0 \\ 16 \\ 2 \end{matrix}$

H	T	O
●	●●●●	●●

How many groups of 6 are in 13 tens? $6 \overline{) 130} \begin{matrix} 0 & 2 \\ 1 & 3 & 2 \end{matrix}$

H	T	O
	●●●●	●●●●

How many groups of 6 are in 12 ones? $6 \overline{) 12} \begin{matrix} 0 & 2 & 2 \\ 1 & 3 & 2 \end{matrix}$

Use short division to divide by a single digit.

$$6 \overline{) 132} \begin{matrix} 0 \\ 2 \\ 2 \end{matrix}$$

$$6 \overline{) 132} \begin{matrix} 0 & 2 \\ 1 & 3 & 2 \end{matrix}$$

$$6 \overline{) 132} \begin{matrix} 0 & 2 & 2 \\ 1 & 3 & 2 \end{matrix}$$

Dividing by a 2-digit number using long division

Use equipment to build numbers from groups.



182 divided into groups of 13. There are 14 groups.

Use bar modelling alongside written division to expose the structure.

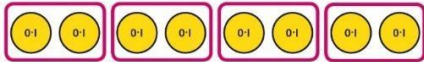
Use long division
Write the required multiples to support the division process.

$$377 \div 13 = ?$$

$$13 \overline{) 377} \begin{matrix} 3 & 7 & 7 \\ - 1 & 3 & 0 & 10 \\ \hline & 2 & 4 & 7 \\ - 1 & 3 & 0 & 10 \\ \hline & & 1 & 7 \\ - 1 & 1 & 7 & 9 \\ \hline & & & 0 \end{matrix}$$

$$377 \div 13 = 29$$

0 29

			<p>A slightly different layout may be used, with the division completed above rather than at the side.</p> $\begin{array}{r} 3 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \end{array}$ $\begin{array}{r} 38 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \\ - 168 \\ \hline 0 \end{array}$ <p>Divisions with a remainder explored in problem-solving contexts.</p>								
Dividing decimals	<p>Use place value equipment to explore division of decimals.</p>  <p>8 tenths divided into 4 groups. 2 tenths in each group.</p>	<p>Use a bar model to represent divisions.</p> <table border="1" data-bbox="965 791 1312 874"> <tr> <td colspan="4" style="text-align: center;">0.8</td> </tr> <tr> <td style="text-align: center;">?</td> <td style="text-align: center;">?</td> <td style="text-align: center;">?</td> <td style="text-align: center;">?</td> </tr> </table> <p> $4 \times 2 = 8$ $8 \div 4 = 2$ So, $4 \times 0.2 = 0.8$ $0.8 \div 4 = 0.2$ </p>	0.8				?	?	?	?	<p>Use short division to divide decimals with up to 2 decimal places.</p> $\begin{array}{r} 0.2 \\ 8 \overline{) 4.24} \\ \underline{8} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$ $\begin{array}{r} 0.5 \\ 8 \overline{) 4.24} \\ \underline{4} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 4 \\ \underline{4} \\ 0 \end{array}$
0.8											
?	?	?	?								

Times tables expectations

Towngate Primary Academy believe that when children are proficient in times tables it enables them to calculate more efficiently.

Expectations within each year group:

Year 1

Children to be able to count in 2's, 10's and 5's

Year 2

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts.

Summer term introduce 3's.

Year 3

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts.

Recall the 4, 8 and 3 multiplication tables, and corresponding division facts.

Summer term to introduce 6's.

Year 4

Recall the 2, 10 and 5 multiplication tables, and corresponding division facts.

Recall the 4, 8 and 3 multiplication tables, and corresponding division facts.

Recall the 6, 9 and 12 multiplication tables, and corresponding division facts.

Recall the 7 and 11 multiplication tables, and corresponding division facts.

Statutory multiplication check in Spring term.

Year 5 & 6

Rapid recall of all times tables and corresponding division facts through continued practice.